

# Design of Reinforced Cement Concrete Structures (3-2-1)

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## Text Books:

1. Jain, A.K. Reinforced concrete: Limit state Design, Roorkee: Nem chand and co.
2. Jain, Krishna and Jain, O.P. Design of R.C.C. structure.

## Chapter:- One Introduction.

### 1.1 Limitation of plain concrete.

Plain cement concrete is a hardened mass obtained from a mixture of cement, sand, gravel and water in definite proportions. These ingredients are mixed together to form a plastic mass which is poured into desired shape molds called as forms.

Plain cement concrete has good compressive strength but very little tensile strength, thus limiting its use in construction. It is used where good compressive strength and weight are the main requirements and tensile stresses are very low.

### Limitations.

- a) plain concrete is quasi-brittle materials.
- b) plain concrete has low tensile strength i.e. tensile strength is  $(\frac{1}{10})$ th of its compressive strength.
- c) concrete has low toughness i.e. only (1-2) % of toughness of steel.
- d) concrete has low specific strength.
- e) concrete has long curing time. i.e. concrete attains specified compressive strength in 28 days.

### 1.2 Properties of reinforcement and concrete.

Types of reinforcement with their properties are as follows:-

- i) Mild steel Reinforcement :- Mild steel bars are also known as Fe-250 because the yield strength of this steel is  $250 \text{ N/mm}^2$ . The stress-strain curve for mild steel is given in figure. It shows a clear, definite yield point. Although they are very ductile, they are not preferred over high yield strength deformed bars because of their less strength.

**WSM**

The design is based on deterministic approach.

The design is governed by elastic theory.

It does not provide a realistic member of the actual factor of safety underlying a design. It is a traditional method of design.

permissible stress is obtained from the yield stress divided by FOS under working load.

**LSM**

The design is based on probabilistic approach.

The design is governed based on limit states of safety and serviceability.

It provides a realistic member of the actual factor of safety. It is modern method of design.

separate partial factor of safety for loads and strength are to be used to get design loads and design strength.

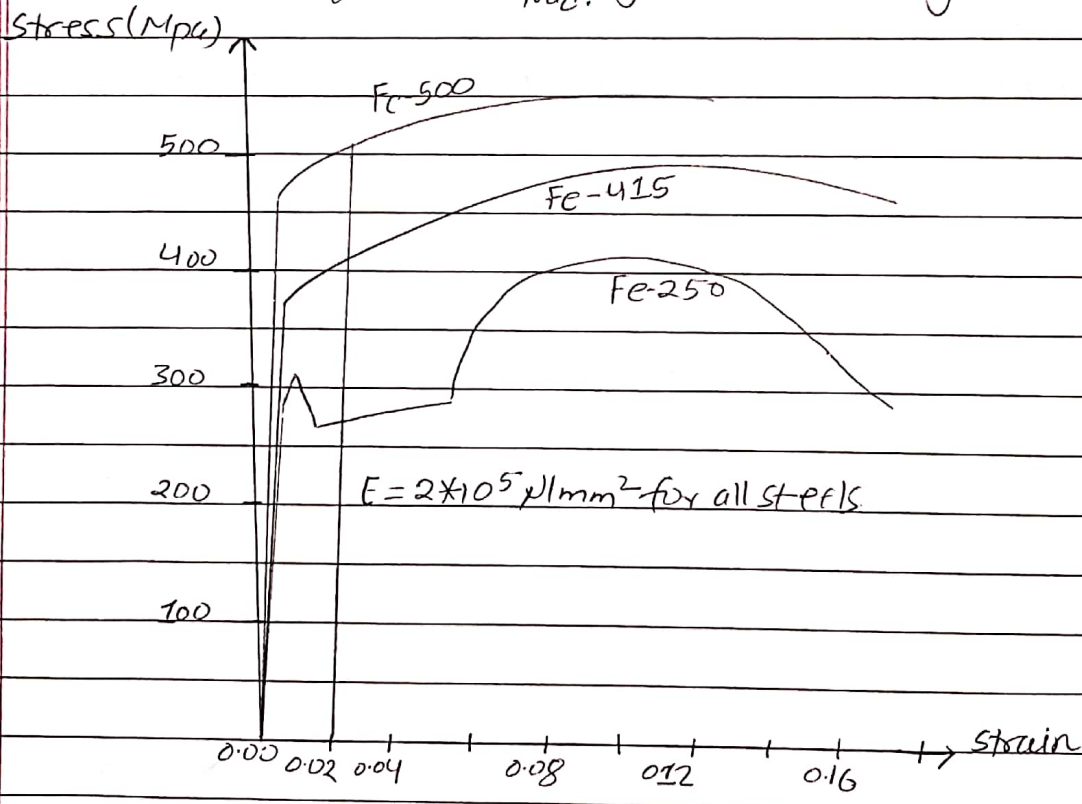


Fig: Typical stress-strain curves for various types of steel.

and weak bond. Its modulus of elasticity is  $2 \times 10^5 \text{ N/mm}^2$ . They are used as lateral ties in columns and at places where nominal reinforcement is required.

ii) High yield strength deformed bars: - They have higher percentage of carbon as compared to mild steel. Its strength is higher than that of mild steel, but the yield point is not clearly defined as shown in figure.

It is available in two types

a) Hot rolled high yield strength bars.

b) Cold worked high yield strength bars.

Cold worked high yield strength bars a.k.a cold Twisted Deformed (CTD) bars or Tor steel are available in two grades

i) Fe 415 or Tor 40      ii) Fe 500 or Tor 50.

A twisted deformed bar has about 50% higher yield stress than plain bars.

HYSD are preferred as reinforcement in R.C.C over plain mild steel bars because of

\* Higher strength: - It has yield strength, higher than that of plain mild steel bars.

\* Better Bond: - It has better bond with concrete due to corrugation on ribs on the surface of the bars.

iii) Thermo-Mechanically Treated (TMT) steel bars: - They are manufactured by passing hot rolled steel bars through cold water. By doing this the outer surface of the bar becomes harder while the inner core is still softer. It has following advantages

\* High yield strength      \* Better weldability

\* Excellent ductility      \* Superior corrosion resistance.

## Properties of concrete.

a) **compressive strength**:- The compressive strength of concrete is determined by the cube test. It is defined as the compressive strength of 150mm cubes of 28 days in  $N/mm^2$  below which not more than 5% of the test samples are expected to fail. It is represented by  $f_{ck}$ . The strength of concrete is greatly affected by water-cement ratio (w/c). As per Abram's law

$$C = \frac{A}{B^x} \quad \text{where } C = \text{compressive strength}$$

A & B are constant  
x = w/c ratio

b) **workability**:- The workability of concrete is defined as the ease with which the concrete can be mixed, handled, placed, compacted & finished. A workable concrete should not bleed or segregate.

workability of concrete depends on.

- ↳ w/c ratio
- ↳ size & shape of aggregate
- ↳ Ratio of fine to coarse aggregate.

c) **durability**:- The concrete should be durable to the environment when it is exposed to the surrounding. It is defined as the ability to resist weathering action, chemical attack, abrasion or any other process of deterioration. A durable concrete will retain its original form, quality and serviceability when exposed to its intended service environment.

d) **Tensile strength**:- The tensile strength of concrete can be correlated with the characteristic compressive strength of concrete. IS 456 gives the following correlation which can be used in the design.

$$f_t = 0.7 \sqrt{f_{ck}}$$

f<sub>1</sub> Modulus of Elasticity:- The term static modulus of elasticity can also be expressed in terms of the characteristic compressive strength and may be written as follows  $E_c = 5000 \sqrt{f_{ck}}$

f<sub>2</sub> Poisson's Ratio:- It is defined as the ratio of lateral strain to the longitudinal strain. It can be taken as 0.2 for all design calculation purposes

g) Creep:- It is defined as the increase in strain of the concrete element with time under sustained load after taking consideration the time-dependent deformation not associated with stress i.e. shrinkage swelling. It depends upon stress level, Age of loading, Duration of loading

Age of loading	Creep coefficient
7	2.2
28	1.6

h) Shrinkage:- Shrinkage in concrete is generally caused by the loss of water through evaporation or by hydration of cement. It is also affected by the fall of temperature and carbonation. Various types of shrinkage are plastic, drying, autogenous, carbonation shrinkage. Factors affecting the shrinkage are

- ↳ effect of w/c ratio
- ↳ type of aggregate
- ↳ relative humidity
- ↳ time.

1.3) Analysis of forces and stresses in reinforced concrete structure. Structures are designed to withstand various types of loads. The various type of loads expected on a structure are as follows.

- ↳ Dead loads
- ↳ live loads
- ↳ wind loads
- ↳ snow loads
- ↳ earthquake loads

• **Dead load** :- They are due to self wt. of structure which are permanent in nature. It depends upon the unit wt. of materials. It includes the self wt. of walls, floors, beams, column etc.

code IS 875 (part-I)-1987

unit wt. of column Building Materials

Materials	unit wt $kN/m^3$
plain cement concrete.	24
Reinforced cement concrete steel.	25
steel	78.5
Brick masonry (Cement plaster)	20

• **Live Loads** :- Live loads on floors and roofs consists of all the loads which are temporarily placed on the structure. These loads keep on changing from time to time. Various types of imposed loads coming on the structure are given in IS 875 (part-2). 1987.

• **Wind Loads** :- The force exerted by the horizontal component of wind is to be considered in the design of buildings. It depends upon the velocity of wind, shape and size of the buildings. His design criteria are provided in IS 875 part-3 1987.

• **Snow Loads** :- The building which are located in the regions where snowfall is common, are to be considered for snow load. The code IS 875 part 4: 1987 deals with snow load on roofs of the building.

• **Earthquake Loads** :- It depends upon the place where the building is located.

## Chapter:- Two Design Methods

### 2.1) Working stress method for design of RCC structures

This method of design was the oldest one. It is based on the elastic theory and assumes that both steel and concrete are elastic and obey Hooke's Law. It means that the stress is directly proportional to strain upto the point of collapse.

#### Assumptions

- ↳ plane section before bending remains plane even after bending.
- ↳ All tensile forces are taken by reinforcement unless otherwise stated.
- ↳ stress strain curve is always linear.
- ↳ The moduli of elasticity of steel ( $E_s$ ) & concrete ( $E_c$ ) are constant.
- ↳ There are no initial stresses in steel and concrete.
- ↳ The modular ratio  $m = \frac{280}{3600}$  (ratio bet<sup>n</sup> elastic moduli of steel & concrete & is denoted by  $m$ )

#### Drawbacks

- ↳ It uses factor of safety for stresses only and not for load.
- ↳ It does not use any factor of safety with respect to loads.
- ↳ It does not account for shrinkage and creep which are time dependent and plastic in nature.
- ↳ This method gives uneconomical sections.
- ↳ It pays no attention to the condition that arise at the time of collapse.

\* Balanced, Under-Reinforced and Over-Reinforced sections as per WSM

a) Balanced section :- A balanced section is that in which concrete and steel reach their permissible value at the same time. The



percentage of steel corresponding to this section is called as balanced steel and the neutral axis is called as critical neutral axis ( $n_c$ )

$$\frac{m \cdot \sigma_{cbc} - n_c}{\sigma_{st} \quad d - n_c}$$

For a balanced section, the moment of resistance is calculated as under

$$M_R = \frac{\sigma_{cbc}}{2} \cdot b \cdot n_c \cdot \left( \frac{d - n_c}{3} \right) = R \cdot b \cdot d^2$$

OR

$$M_R = \sigma_{st} \cdot A_{st} \cdot \left( \frac{d - n_c}{3} \right)$$

b) Under-Reinforced section: In an under-reinforced section, the percentage of steel provided is less than that provided in balanced section. So, the actual neutral axis will shift upward i.e.  $n_c > n$ . The moment of resistance of this section is calculated as

$$M_R = \sigma_{st} \cdot A_{st} \cdot \left( \frac{d - n}{3} \right)$$

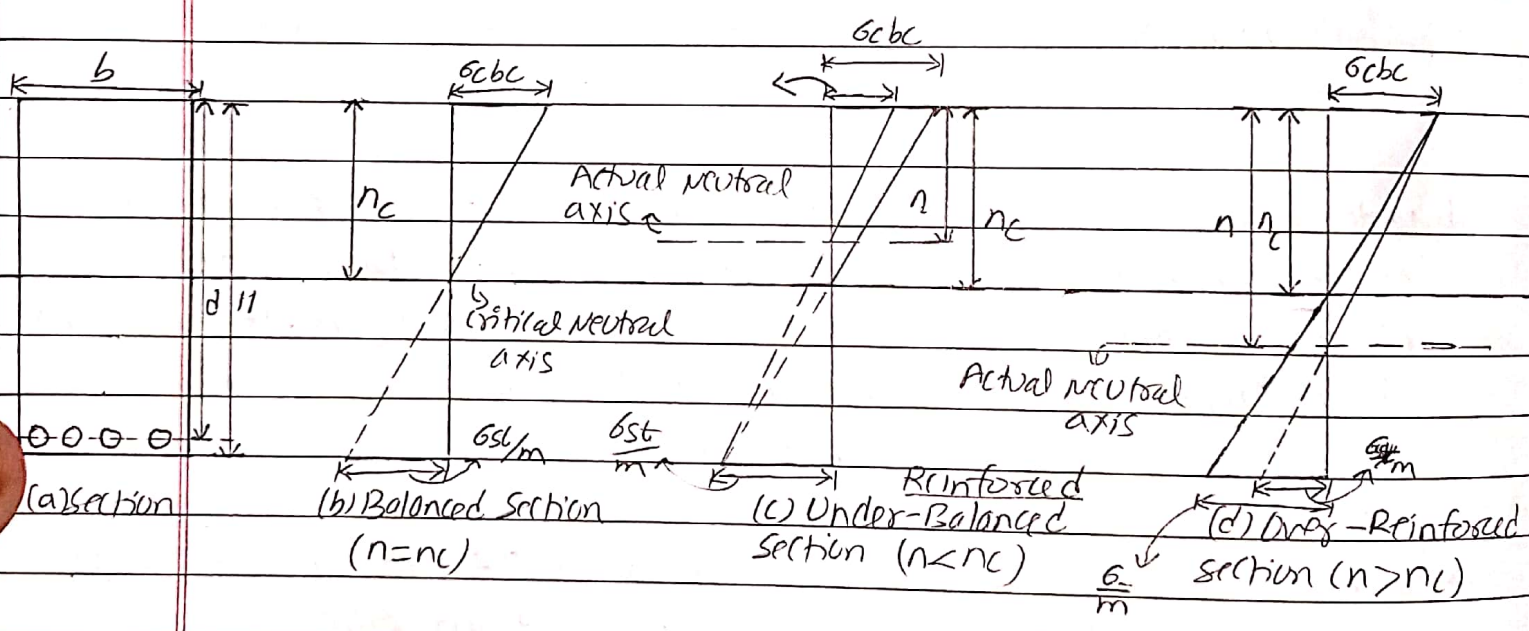
Features

- \* Steel is fully stressed while concrete is not i.e. stress in steel is  $\sigma_{st}$  (permissible) but stress in concrete is less than  $\sigma_{cbc}$ .
- \* The actual neutral axis lies above the critical neutral axis ( $n_c > n$ ).
- \* Ductile failure.
- \* The moment of resistance is less than balanced section.

c) Over-Reinforced sections: In an over-reinforced section the percentage of steel provided is greater than the balanced section. So the actual neutral axis shift downwards i.e.  $n > n_c$ . As steel is not fully utilized, the section is uneconomical.

### Features

- \* concrete is fully stressed while steel is not (i.e. the stress in concrete is at its permissible value  $\sigma_{cbc}$  but stress in steel is less than  $\sigma_{st}$ )
- \* The actual neutral axis is below the critical neutral axis. i.e.  $n > n_c$ .
- \* Sudden failure.
- \* The moment of resistance of over-reinforced section is calculated as  $M_r = \frac{1}{2} \sigma_{cbc} \cdot b \cdot n \left( \frac{d-n}{3} \right)$



### Design procedure Reinforced beam

- (1) For the given grade of concrete and steel, determine the permissible stresses i.e.  $\sigma_{cbc}$  &  $\sigma_{st}$  from code IS 456:2000 table 21 and 22
- (2) Calculate modular ratio  $m = \frac{280}{\sigma_{cbc}}$
- (3) Determine critical neutral axis ( $n_c$ ). 
$$m \cdot \sigma_{cbc} = n_c$$
  
$$\sigma_{st} \quad d - n_c$$

(4) Determine the actual neutral axis  $\frac{b \cdot n^2 - m \cdot A_{st} \cdot (d - n)}{2}$

(5) compare  $n$  &  $n_c$

i) If  $n = n_c$ , the section is balanced and the moment of resistance can be calculated by any of the following equation.

$$M_B = \frac{6bc \cdot b \cdot n_c \cdot (d - n_c)}{3} = R_b \cdot d^2$$

OR

$$M_B = 6st \cdot A_{st} \cdot \left(\frac{d - n_c}{3}\right)$$

ii) If  $n < n_c$  the section is under reinforced and the moment of resistance is calculated as

$$M_x = 6st \cdot A_{st} \left(\frac{d - n}{3}\right)$$

iii) If  $n > n_c$ , the section is over-reinforced and the moment of resistance is calculated as

$$M_x = \frac{1}{2} 6bc \cdot b \cdot n \cdot \left(\frac{d - n}{3}\right)$$

⊗ Sometimes it is required to find out the safe load ( $w$ ) which the beam can carry. For this, the maximum bending moment due to the loads is calculated and equated to the moment of resistance of the section.

The maximum bending moment value for some beams are :-

a) simply supported beam for u.d.l. =  $\frac{wl^2}{8}$  (sagging),  $wl$  (point load)  
or  $w \cdot a \cdot b$

b) cantilever beam for u.d.l. =  $\frac{wl^2}{2}$  (hogging),  $wl$  (point load)

where  $l$  is the effective span of the beam

## 2.3 Ultimate load method for design of RCC structures

In this method ultimate or collapse load is used as design load. The ultimate loads are obtained by increasing the working loads suitably by some factors. These factors which are multiplied by the working loads to obtain ultimate loads are called as load factors. This method uses the real stress-strain curve of concrete and steel and takes into account the plastic behavior of these materials.

$$\text{Load factor} = \frac{\text{collapse load}}{\text{working load}}$$

The load factors provide a clear margin of safety and one can easily tell the load at which the structure fails, which is not clear from the working stress concept of permissible.

### Advantages

- ↳ This method is more realistic as compared to working stress method because ultimate load method takes into account the non-linear behavior of the concrete.
- ↳ This method gives exact margin of safety in terms of load unlike working stress method which is based on the permissible stresses.
- ↳ This method is economical as compared to WSM.

### Limitation

- ↳ This method gives very thin sections which leads to excessive deformations and cracking.
- ↳ No factors of safety are used for material stresses.

As serviceability requirements are not satisfied, limit state method are used which takes into account the strength as well as serviceability requirements.

## 2.3) Limit state method for design of RCC structures

This is the most rational method which takes into account the ultimate strength of the structure and also the serviceability requirements. It is a judicious combination of working stress and ultimate load method of design. This method is based on the concept of safety at ultimate loads (ultimate load method) and serviceability at working load (working stress method). The two important limit states to be considered in design are  $\rightarrow$  limit state of collapse  $\rightarrow$  limit state of serviceability.

## 2.4) Types of limit state methods

### 1) Limit state of collapse.

The limit state corresponds to the strength of the structure and categorized into following types.

- limit state of collapse  $\rightarrow$  flexure
  - $\rightarrow$  shear and bond
  - $\rightarrow$  torsion
  - $\rightarrow$  compression.

From code IS 456:2000. CL 35.7  
Pg 69

### 2) Limit state of serviceability

The limit state corresponds to the serviceability requirements i.e. deflection, cracking and vibration.

#### A) Deflection control

a) factors affecting Deflection

- Magnitude of load and their distribution.
- span and type of span.
- cross-sectional characteristics of structural member.
- Types of concrete.
- stress in steel reinforcement.

Amount and extent of cracking

b) Methods of deflection control:

i) Theoretical method

$$S \leq S_{\text{permissible}}$$

ii) Empirical method (method of sufficient stiffness)

$$l \leq \alpha \cdot \beta \cdot \gamma \cdot S \cdot \lambda \quad (\text{Cl. 32.2.1 code IS 456:2000 p.937})$$

where

$l$  = effective length,  $d$  = effective depth.

$\alpha$  = basic value (boundary condition) (23.2.1-(a))

$\beta$  = depend upon length of member (23.2.1-(b))

$\gamma$  = depend upon tensile R reinforcement (23.2.1-(c))

$S$  = depend upon shape of member

$\lambda$  = depend upon compressive reinforcement

B) CRACK CONTROL

a) causes of cracks

- Due to settlement of plastic concrete.
- Due to uneven volumetric change of concrete.
- Due to external loading

b) Methods of crack control

i) Theoretical method (Annex F IS 456:2000 p.95)

$$\Delta_c \leq \Delta_{\text{per}}$$

$\Delta_c$  = calculated crack width.

$\Delta_{\text{per}}$  = permissible crack.

ii) Empirical method (rule of proper detailing)

(cracking is controlled by proper detailing)

## 2.5 characteristics loads and strength of materials

### characteristics strength of materials

The term characteristics strength means that value of the strength of the materials below which not more than 5 percent of the test results are expected to fall.

C.I. 36.1 of IS 456-2000 p. 967.

### characteristics load

The term characteristics load means that value of load which has 95% probability of not being exceeded during the life of the structure

C.I. 36.2 of IS 456-2000 p. 967

## 2.6 partial safety factors and their considerations in structural design.

### a) Materials:-

The design strength of the materials  $F_d$  is given by

$$F_d = \frac{F}{\gamma_m}$$

C.I. 36.3 IS 456-2000  
p. 968

where  $F$  = characteristic strength of the material  $\rightarrow$  C.I. 36.1 IS 456-2000  
p. 967

$\gamma_m$  = partial safety factor appropriate to the material and the limit state being considered. 1.5 for concrete & 1.15 for steel.

### b) Loads

The design load  $F_d$  is given by

$$F_d = F \gamma_f$$

where  $F$  = characteristics load  $\rightarrow$  C.I. 36.2 IS 456-2000 p. 967

$\gamma_f$  = partial safety factor appropriate to the nature of loading and the limit state being considered.

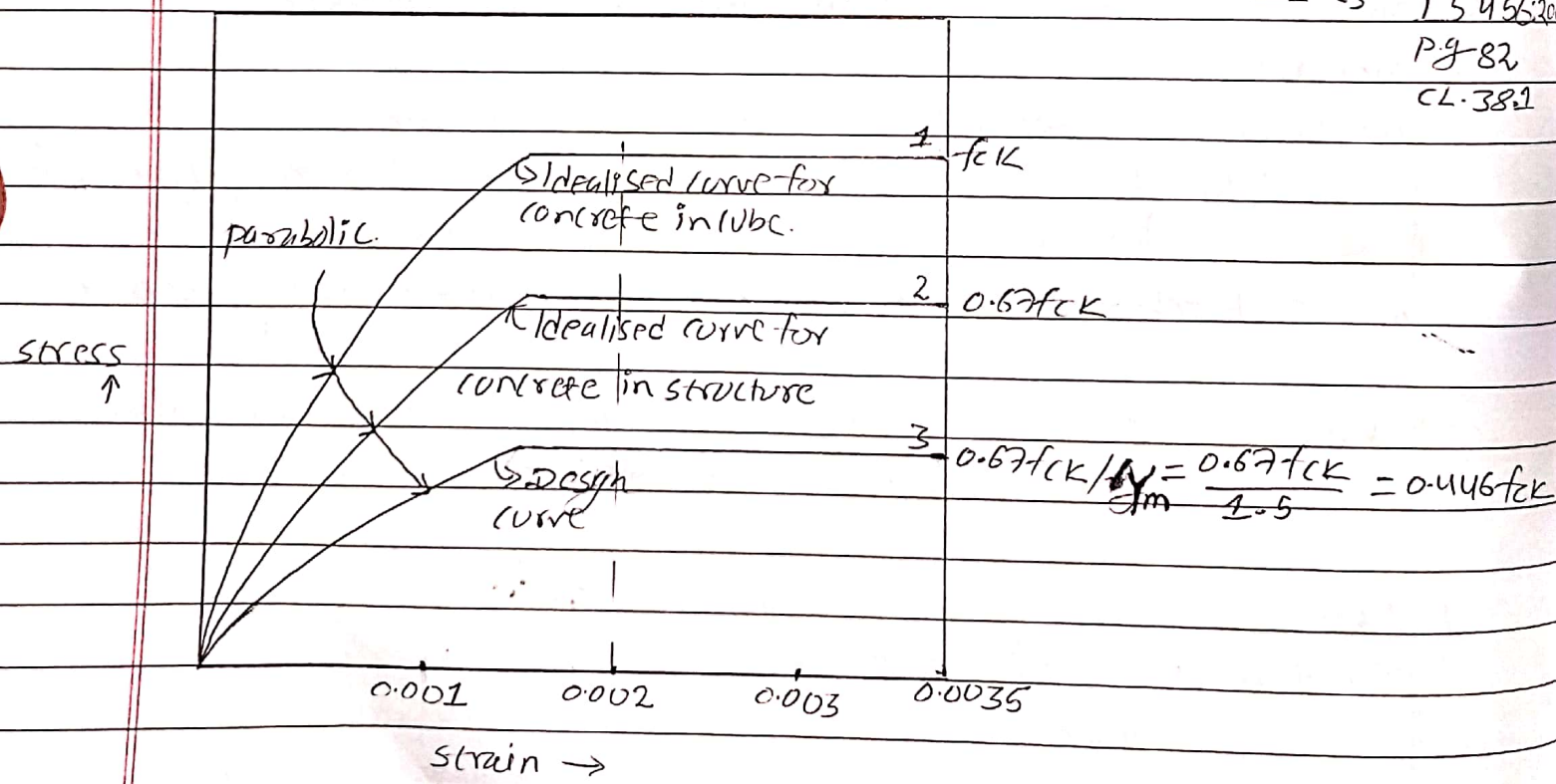
# Chapter: Three Limit state Design for Beams and slab.

## 3.1 General design consideration.

### Basic Assumption.

- plane section normal to the axis of the member remains plane after bending.
- The maximum strain in concrete at the outermost compression fiber is 0.0035.
- The tensile strength of concrete is ignored.
- The strain in the tension reinforcement is to be not less than  $\frac{0.87f_y}{E_s} + 0.002$

or,  $f_y + 0.002$   
 $\frac{1.15 E_s}{T 5 456:200}$   
 Pg 82  
 CL-381





## Chapter 1 Remaining.

Plain cement concrete (P.C.C.) :- concrete is a composite material composed of fine & coarse aggregate combined with fluid cement that hardens over time. When aggregate is mixed with dry cement and water it forms a cement slurry which is easily poured and moulded into shape. Cement reacts chemically with water and other ingredients to form a complex matrix which binds all the material together to form a strong durable stone like material which has many uses.

Reinforced cement concrete (R.C.C.) :- concrete is a mixture of cement, fine aggregate, coarse aggregate (crushed stone) and water, the behaviour of concrete is that it is very strong in compression but weak in tension. Steel behaves opposite to that of concrete i.e. it is very strong in tension and weak in compression. When concrete or steel alone is used, it is not able to withstand all compressive and tensile forces. So, concrete and steel alone is used, it is not able to withstand all compressive and tensile forces. So, concrete and steel bar are combinedly used to form a structural element. So that it will be strong and safe against all forces. Such structural elements are called reinforced cement concrete (R.C.C. structures).

R.C.C. structure = compressive force of concrete + tensile force of steel.

properties of P.C.C./advantages.

- strength & durability

- Affordability

- Low maintenance cost.

- Locally produced and used

- Fire resistant

Energy efficient in production.

- Albedo (reflection of heat & light) i.e. it doesn't absorb energy and hence temperature does not increase.

### Limitation of R.C.C.

- Concrete is a brittle material.
- It has low toughness, low tensile strength.
- It has low specific strength.
- Formwork is required.
- Long curing time.
- Working with cracks
- Demands strict quality control

### Chapter :- 2, Remaining

#### Methods of design of R.C.C. structures

- Working stress method (WSM)
- Ultimate stress method (USM)
- Limit stress/state method (LSM)

#### 1) Working stress method (WSM)

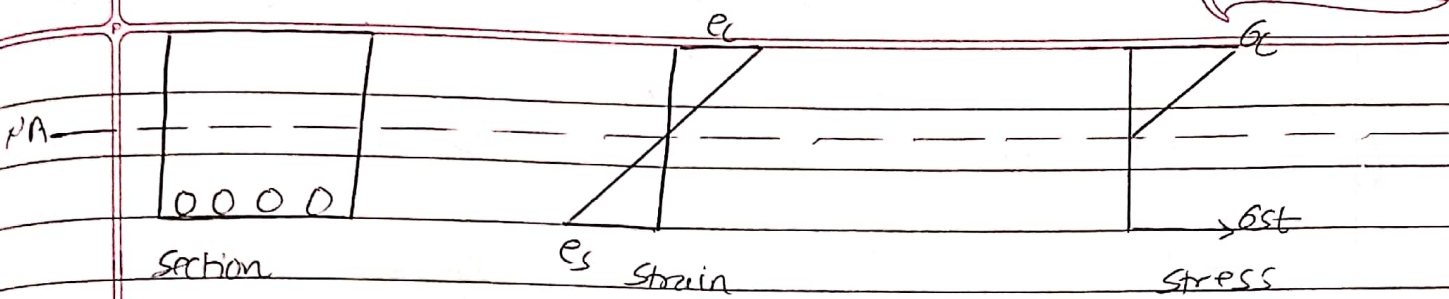
It is the traditional method of design where it is assumed that concrete is elastic and concrete and steel both acts together elastically.

The relationship between loads stress is linear upto the collapse of the structure. The basis of this method is that the permissible stresses for concrete and steel are not exceeded anywhere in the structure even in the application of worst combination of load. The members are designed in accordance with elastic theory of bending assuming both concrete and steel obeys Hooke's law. It assume linear variation of stress and strain.

Permissible stress is provided by assuming suitable factor of safety to allow for the uncertainties in the estimation of working load and variation in strength of material.

concrete  $f_{os} = 1.5$

Steel  $f_{os} = 1.15$



i.e.  $\mu R \geq L$ ,  $R$  = Resistance of material.

$L$  = Working load.

$\mu$  = inverse of factor of safety  
less than 1 ( $\mu < 1$ )

### Drawbacks

- concrete is not elastic
- Factor of safety is provided only to the strength of material
- Difficult to consider creep and strength.

### 2) Limit state method (LSM)

It is derived from plastic theory of structure. The object of design based on limit state concept is to achieve an acceptable probability that a structure will not deform on service in its life times for use which it is intended i.e. will not reach limit state.

The limit state concept of design of R.C.C structure takes into account of the probabilistical and structural variation in material properties, loads, and safety factors. LSM design can be expressed as

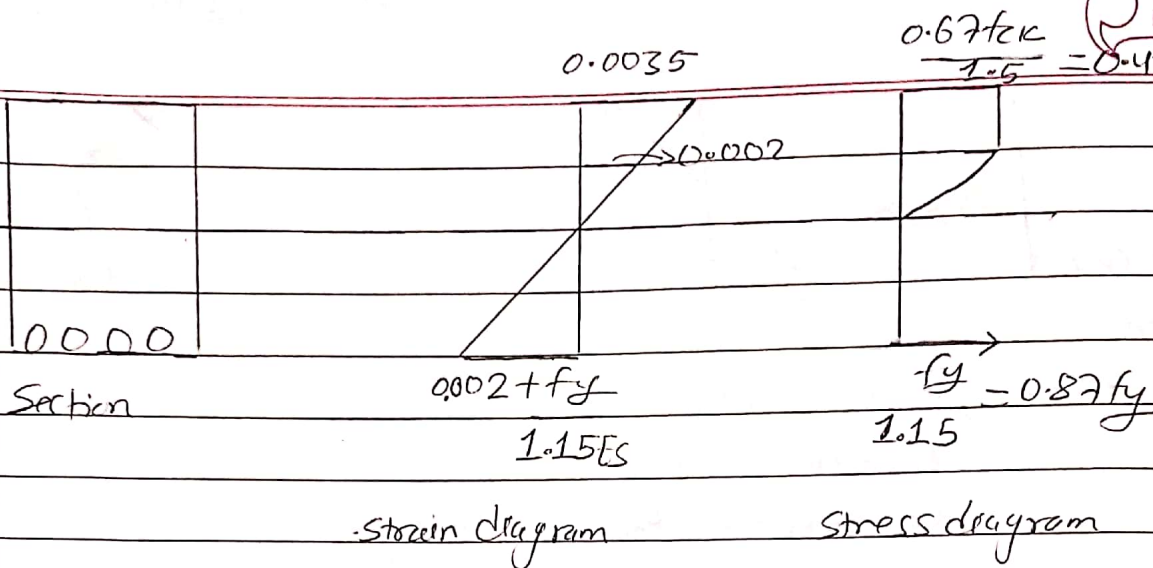
$$\mu R > \sum_{i=1}^n \lambda_i L_i$$

$R$  = resistance of material       $\mu \lambda =$  safety factors for material and load.  
 $L$  = working load

LSM assume that maximum compressive strength in outermost fiber of concrete is  $0.35 f_{ck}$  and maximum tensile strength in steel is not less than  $0.002 t f_y$

$$1.15 E_s$$

$f_y$  = yield stress of steel.  
 $E_s$  = young's modulus of elasticity



partial factor of safety is considered to account for variation and uncertainties for the strength of material and calculation of loads.

Types of limit states.

- Limit state of collapse :- It is concerned with safety of people and safety of structure that all the loads acting on the structure can be withstood by given section. It is concerned with
  - flexure or tension
  - Shear
  - compression
  - torsion.

Ultimate limit state for collapse.

- loss of equilibrium
- excessive deformation.
- Rupture
- Loss of stability
- fatigue.

• Limit state of serviceability :- This state is concerned with the functioning of the structure under normal use or comfort of people's appearance of structure. It corresponds to

- deformation that affects appearance, comfort of user & functioning of structure.
- vibration that correspond to people's functional effectiveness
- Damage that is likely to affect appearance durability, functioning of structure.

Limit state for serviceability considering deflection can be expressed as  $S \leq \frac{L}{\lambda}$   
 $S$  = deflection       $L$  = span length.  
 $\lambda$  = non-dimensional number.

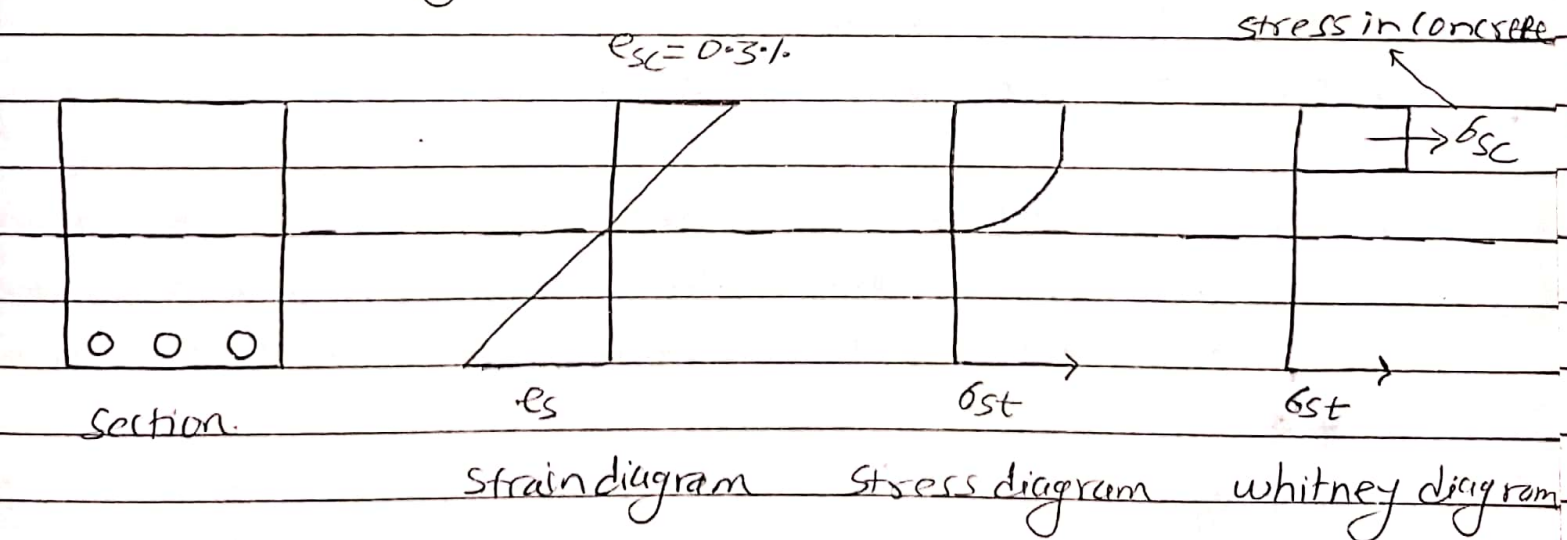
For calculation of deflection only, service loads are used.

### Ultimate state method (USM)

In USM, loads are increased by suitable factor and structure is designed to withstand ultimate load. The factor is called load factor. This method uses the non-linear behavior of concrete.

Load factor =  $\frac{\text{Ultimate load or collapse load}}{\text{working load}}$

For ultimate load method, mostly Whitney's principle is used which assumes maximum <sup>compressive</sup> strength in concrete is 0.3% and stress corresponding to strain.



Ultimate state method can be expressed as  $R > \lambda L$

where

$R$  = resistance of material       $\lambda$  = Load factor.

$L$  = working load

## Drawbacks

- factor of safety is only provided for loads.
- complete disregard for control of deflection.

## Partial factor of safety

- ⊗ Partial factor of safety for material,  $M_m$
- ⊗ Partial factor of safety for load,  $M_L$

⊗ Partial factor of safety for material ( $M_m$ ):- It is the factor incorporated to account for the possible unfavorable deviation from the strength of material, from the characteristic value, from the possible unfavourable deviation of sectional dimension, accuracy of calculation procedure and risk to the life and economy.

- \* Partial factor of safety for concrete = 1.5
- \* Partial factor of safety for steel = 1.15

⊗ Partial factor of safety for load ( $M_L$ ):- It is the factor which is taken into consideration while calculating load to account for

- ↳ unusual increase in load beyond that used for deriving characteristic value.
- ↳ unforeseen stress redistribution
- ↳ inaccurate assessment of the effect of loading.

Characteristic strength ( $f_{ck}$ ) :- It is the strength below which not more than 5% of the test results are expected to fail.

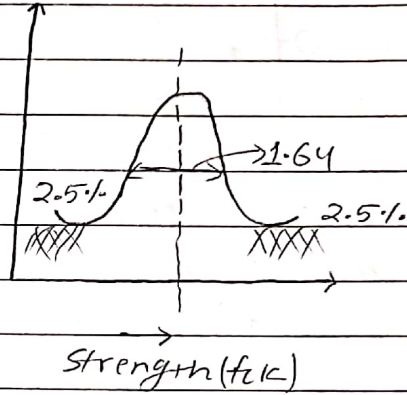
characteristic strength

$$f_{ck} = \bar{x} + 1.64\sigma$$

↑  
frequency

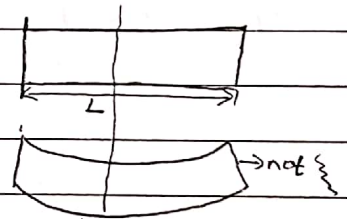
$\bar{x}$  = mean strength of sample

$\sigma$  = Standard deviation.



Characteristic load ( $w_{ck}$ ) :- The load which has 95% expectation of not exceeding during the life time of the structure.

$$w_{ck} = \bar{x} + 1.64\sigma$$



Limit state method (LSM)

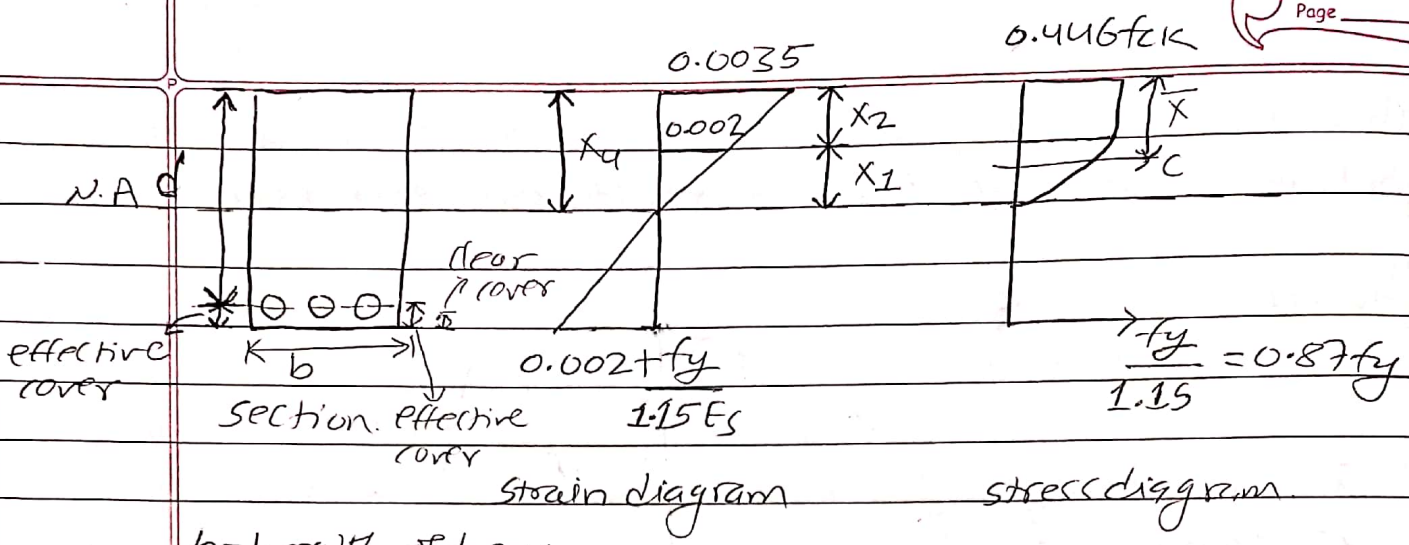
Basic assumption

- 1) plane section normal to the axis of beam remains plane even after bending.
- 2) Tensile strength of concrete is neglected.
- 3) stress-strain diagram for concrete is rectangular-parabolic with maximum compressive at outermost fibre of concrete is

$$\frac{0.67 f_{ck}}{1.5} = 0.446 f_{ck}$$

- 4) Maximum compressive strain at outermost fibre of concrete is taken as 0.35%.

- 5) maximum tensile strain in steel at ultimate limit is taken as not less than  $0.002 + \frac{f_y}{1.15 E_s}$



$b =$  breadth of beam.

$d =$  effective depth (distance from extreme fiber of centre to the centre of tension reinforcement).

$x_u =$  depth of neutral axis.

From similar  $\Delta$ 's

$$\frac{x_1}{x_u} = \frac{0.002}{0.0035} = \frac{4}{7}$$

$$\text{or, } x_1 = \frac{4x_u}{7}$$

$$x_2 = x_u - x_1 = x_u - \frac{4x_u}{7} = \frac{3x_u}{7}$$

compressive force of rectangular block  $C_2 = 0.446 f_{ck} \cdot x_2 \cdot b$

$$= 0.446 f_{ck} \cdot \frac{3x_u}{7} \cdot b$$

$$= 0.191 f_{ck} b x_u$$

compressive force of parabolic block  $C_1 = \frac{3}{3} \times 0.446 f_{ck} \cdot x_1 \cdot b$

$$= \frac{2}{3} \times 0.446 f_{ck} \times \frac{4x_u}{7} \cdot b$$

$$= 0.17 f_{ck} \cdot x_u \cdot b$$



parabola  $\bar{x} = \frac{3a}{8}$

Total compressive force of concrete,  $C = C_1 + C_2 = 0.17 f_{ck} \cdot b \cdot X_u + 0.19 f_{ck} \cdot b \cdot X_u$   
 $C = 0.36 f_{ck} \cdot b \cdot X_u$

To find the line of action of total compressive force  $C$

$$\bar{X} = \frac{C_1 \bar{X}_1 + C_2 \bar{X}_2}{C_1 + C_2} = \frac{C_1 (X_u + \frac{3X_u}{8}) + C_2 \frac{X_u}{2}}{C_1 + C_2}$$

$$= 0.17 f_{ck} \cdot b \cdot X_u \left( \frac{3X_u}{8} + \frac{3 \cdot 4X_u}{7 \cdot 8} \right) + 0.19 f_{ck} \cdot b \cdot X_u \cdot \frac{3X_u}{7 \cdot 2}$$

$$0.17 f_{ck} \cdot b \cdot X_u + 0.19 f_{ck} \cdot b \cdot X_u$$

$$= f_{ck} \cdot b \cdot X_u^2 \left( \frac{0.17 \times 9}{14} + \frac{0.19 \times 1.5}{7} \right)$$

$$0.36 f_{ck} \cdot b \cdot X_u$$

$\frac{3 + 3 \times 4 - 9}{7 \cdot 7 \cdot 8 \cdot 14}$
$\frac{3 - 1.5}{7 \cdot 2 \cdot 7}$

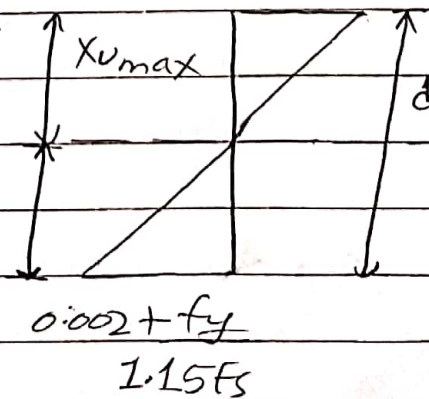
$$= \frac{0.15}{0.36} X_u = 0.4166 X_u \approx 0.417 X_u$$

To find the neutral axis depth for balanced section or limiting depth of neutral axis,  $X_{u,max}$

From similar  $\Delta$ 's

$$\frac{d - X_{u,max}}{X_{u,max}} = \frac{0.002}{1.15 E_s} \quad d - X_{u,max} = \frac{0.002 X_{u,max}}{1.15 E_s}$$

$$d - X_{u,max} = \frac{0.002 X_{u,max}}{1.15 E_s}$$



$$0.002 + f_y = 1.15 E_s$$

$$\text{or, } \frac{d}{X_{u,max}} - 1 = \frac{0.002 + 0.87 f_y}{1.15 E_s}$$

$$0.0035$$

$$d - 0.0055 + 0.87 f_y / E_s$$

$$x_{u,max} \frac{0.0035}{0.0055 + 0.87 f_y / E_s}$$

$$\therefore x_{u,max} = \frac{0.0035 d}{0.0055 + 0.87 \frac{f_y}{E_s}}$$

for

for Fe 250,  $x_{u,max} = \frac{0.0035 d}{0.0055 + 0.87 \times 250 / 2 \times 10^5} = 0.53 d$

For Fe 415,  $x_{u,max} = \frac{0.0035 d}{0.0055 + 0.87 \times 415 / 2 \times 10^5} = 0.48 d$

For Fe 500,  $x_{u,max} = \frac{0.0035 d}{0.0055 + 0.87 \times 500 / 2 \times 10^5} = 0.46 d$  From code page 33

To find the depth of neutral axis,  $x_u$

Total compressive force = total tensile force

$C = T$

or,  $0.36 f_{ck} \cdot b \cdot x_u = 0.87 f_y \cdot A_{st}$

or,  $x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot b}$

$\bar{x} = 0.42 x_u$

$C = 0.36 f_{ck} \cdot b \cdot x_u$

$T = 0.87 f_y \cdot A_{st}$

$z = \text{lever arm}$

$A_{st} = \text{Area of steel}$

Lever arm ( $z$ ) = distance between total compressive force of concrete and tensile force of steel

$$= d - \bar{x}$$

$$= d - 0.42 x_u$$

Moment of resistance (MOR) in term of concrete =  $C \cdot z = 0.36 f_{ck} \cdot b \cdot x_u \cdot (d - 0.42 x_u)$

MOR in term of steel =  $T \cdot z = 0.87 f_y A_{st} (d - 0.42 \times 0.87 f_y A_{st})$   
 $0.36 f_{ck} \cdot b$   
 $= 0.87 f_y A_{st} (d - \frac{f_y A_{st}}{f_{ck} \cdot b})$

Percentage of steel := (P<sub>t</sub>) =  $\frac{A_{st}}{b \cdot d} \times 100\%$  =  $\frac{0.36 f_{ck} \cdot b \cdot x_u \times 100\%}{0.87 f_y \cdot b \cdot d}$   
 $\rightarrow$  effective depth =  $\frac{0.36 \times f_{ck} \cdot x_u \times 100\%}{0.87 f_y d}$

Minimum percentage of steel =  $0.85 \cdot \frac{b \cdot d}{f_y}$

maximum percentage of steel =  $4\%$  of  $b \cdot D$   $\rightarrow$  overall depth.  
 $= 0.04 b D$

code page 16 C.1.26.5.1.2

Moment of resistance (MOR) in term of concrete =  $(Z = 0.36 f_{ck} \cdot b \cdot X_u \cdot (d - 0.42 X_u))$

MOR in term of steel =  $Z = \frac{0.87 f_y A_{st} (d - 0.42 \times 0.87 f_y A_{st})}{0.36 f_{ck} \cdot b}$   
 $= 0.87 f_y A_{st} \left( d - \frac{f_y A_{st}}{f_{ck} \cdot b} \right)$

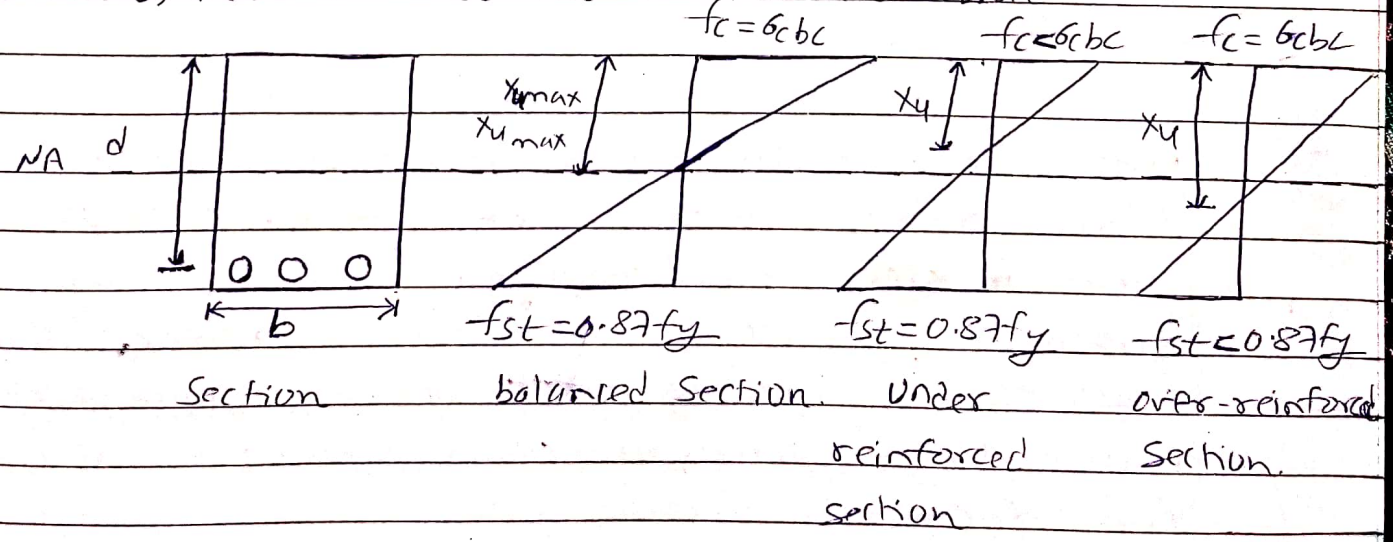
Percentage of steel: -  $(P_t) = \frac{A_{st} \times 100\%}{b \cdot d}$   $= \frac{0.36 f_{ck} \cdot b \cdot X_u \times 100\%}{0.87 f_y \cdot b \cdot d}$   
 $\rightarrow$  effective depth  $= \frac{0.36 \times f_{ck} \cdot X_u \times 100\%}{0.87 f_y d}$

Minimum percentage of steel =  $\frac{0.85 \cdot b \cdot d}{f_y}$

maximum percentage of steel =  $4\%$  of  $b \cdot D$   $\rightarrow$  overall depth.  
 $= 0.04 b D$

code page 16 C.L 26.5.1.7

Balanced, under reinforced and over reinforced section



For balanced section,  $X_u = X_{u,max}$   
 For under-reinforced section,  $X_u < X_{u,max}$   
 For over-reinforced section,  $X_u > X_{u,max}$  (always avoid design of over-reinforced section)

Balanced section is a section at which area of tension steel is such that at which ultimate limit state, i.e. limiting conditions reach simultaneously i.e. tensile strain in steel reaches yield strain which compressive strain at outermost fiber of concrete reaches ultimate strain. Similarly, permissible tensile stress in steel and permissible compressive stress at outermost fibre of concrete reaches simultaneously.

Failure of balanced section is expected with simultaneous initiation of crushing of concrete (brittle failure) and yielding of steel (ductile failure).

Under reinforced section is a section at which area of tension steel is such that at ultimate limit state, tensile strain in steel reaches yield strain before compressive strain at outermost fiber of concrete reaches ultimate strain. Similarly, permissible tensile stress in steel reaches before permissible compressive stress at outermost fiber of concrete reaches.

Failure of under-reinforced section is ductile in nature due to the failure of steel by yielding.

Over-reinforced section is a section at which area of tension steel is such that at ultimate limit state, compressive strain at outermost fibre of concrete reaches ultimate strain before tensile strain at steel reaches yield strain. Similarly, permissible compressive stress at outermost fibre of concrete reaches before permissible tensile stress at steel reaches.

Failure of over-reinforced section is brittle in nature due to the failure of concrete by crushing.

\* Numericals

1. Determine moment of resistance of section as shown. Use M25 concrete and Fe 250 steel.

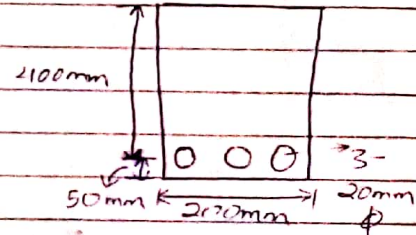
Sol<sup>n</sup>

$b = 200\text{mm}$   
 effective depth ( $d$ ) =  $400\text{mm}$

$f_y = 250\text{MPa}$

$f_{ck} = 25\text{MPa}$

$$A_{st} = \frac{3 \times \pi \times d^2}{4} = \frac{3 \times \pi \times 20^2}{4} = 942.478\text{mm}^2$$



Limiting depth of neutral axis,  $X_{u,max} = 0.53d$  for Fe 250  
 $= 0.53 \times 400$   
 $= 212\text{mm}$

Depth of neutral axis,  $x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$

$$= \frac{0.87 \times 250 \times 942.478}{0.36 \times 15 \times 200}$$

$$= 189.805 \text{ mm} < x_{u, \max}$$

(under-reinforced section)

Moment of resistance in terms of concrete, MOR

$$= 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

$$= 0.36 \times 15 \times 200 \times 189.805 \times (400 - 0.42 \times 189.805)$$

$$= 65654394.51 \times 10^{-6} \text{ kNm}$$

$$= 65.654 \text{ kNm}$$

Interpret steel, MOR =  $0.87 f_y A_{st} (d - 0.42 x_u)$

$$= 0.87 \times 250 \times 942.478 \times (400 - 0.42 \times 189.805) \times 10^{-6}$$

$$= 65.654 \text{ kNm}$$

dia ( $\phi$ )	Area ( $A_{st}$ )
8mm	$\frac{\pi d^2}{4}$ 50.265 mm <sup>2</sup> , 50 mm <sup>2</sup>
10mm	78.539 mm <sup>2</sup> , 78 mm <sup>2</sup>
12mm	113.097 mm <sup>2</sup> , 113 mm <sup>2</sup>
16mm	201.061 mm <sup>2</sup> , 201 mm <sup>2</sup>
20mm	314.159 mm <sup>2</sup> , 314 mm <sup>2</sup>
25mm	490.874 mm <sup>2</sup> , 490 mm <sup>2</sup>
32mm	804.247 mm <sup>2</sup> , 840 mm <sup>2</sup>
28mm	615.75 mm <sup>2</sup> , 615 mm <sup>2</sup>

2 Design a rectangular beam to resist bending moment equal to 45 kNm use M15 mix & Fe 415 steel.

Soln

Given,  $f_y = 415 \text{ MPa}$ ,  $f_{ck} = 15 \text{ MPa}$

Bending moment (M) = 45 kNm

Assume  $d/b = 2$  Factored Bending moment (M) =  $45 \times 1.5 = 67.5 \text{ kNm}$

Limiting depth of neutral axis,  $x_{u, \max} = 0.48d$  for Fe 415

For economy, design balanced section, Now,

MOR =  $0.36 f_{ck} b x_u (d - 0.42 x_u)$ ,  $x_u = x_{u, \max} = 0.48d$

or,  $15 \times 45 \times 10^6 = 0.36 \times 15 \times b \times 0.96b (2b - 0.42 \times 0.96b) = 0.48 \times 2b$

on solving,  $b = 201.278 \text{ mm}$

adopt (b) = 205 mm &  $d = 2b = 410 \text{ mm}$

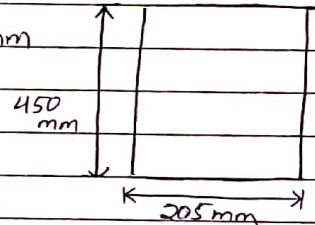
Assume, effective cover 40mm, overall depth (D) =  $410 + 40 = 450 \text{ mm}$

To find area of steel,  $M = \frac{0.87 f_y A_{st} (d - \frac{f_y A_{st}}{f_{ck} b})}{f_{ck} b}$

or,  $67.5 \times 10^6 = \frac{0.87 \times 415 \times A_{st} \times (410 - \frac{415 A_{st}}{15 \times 205})}{15 \times 205}$

on solving

$A_{st} = 558.756 \text{ mm}^2$



Generally effective cover = clear cover +  $d/2 = 35 \text{ to } 40 \text{ mm}$

provide 3-16mm $\phi$  rebar

$$(A_{st})_{provided} = 3 \times 201 = 603 \text{ mm}^2 > 558.756 \text{ mm}^2$$

Minimum  $A_{st}$  to be provided  $(A_{st})_{min} = 0.85 \frac{b \cdot d}{f_y}$

$$= \frac{0.85 \times 205 \times 410 \text{ mm}^2}{415}$$

$$= 172.151 \text{ mm}^2$$

max<sup>m</sup>  $A_{st}$  to be provided,  $(A_{st})_{max} = 0.046 \cdot D$

$$= 0.046 \times 205 \times 460 \text{ mm}^2$$

$$= 3690 \text{ mm}^2$$

check for neutral axis depth,  $x_u = 0.87 \frac{f_y A_{st}}{f_{ck} b}$   $\rightarrow$  Always use provided  $A_{st}$

$$= \frac{0.87 \times 415 \times 603}{0.36 \times 25 \times 410}$$

$$= 196.667 < x_{u,max}$$

$$= 196.667 < x_{u,max} \text{ or } 410$$

$$x_{u,max} = 0.48 \cdot d = 0.48 \times 410 = 196.8 \text{ mm}$$

3) Design a rectangular beam for 4m effective length subjected to dead load of 15 kN/m and live load 12 kN/m. Use M25 mix and Fe500 steel

Given,

effective length,  $(l_e) = 4\text{m}$

$f_{ck} = 25 \text{ N/mm}^2$ ,  $f_y = 500 \text{ N/mm}^2$

assume  $\frac{d_e}{d} = 10 \Rightarrow d = \frac{d_e}{10} = \frac{4000}{10} = 400 \text{ mm}$

Generally

$$\frac{d_e}{d} = \frac{l_e - 30}{d}$$

$$d/b = 2 \Rightarrow b = \frac{d}{2} = \frac{400}{2} = 200 \text{ mm}$$

calculation of load.

live load = 12 kN/m

dead load = 15 kN/m

Self wt =  $25 \times 0.2 \times 0.4 \text{ kN/m}$  {tentative figure}

$$= 2 \text{ kN/m}$$

Total load = 29 kN/m

factored load  $(w_u) = 1.5 \times 29 = 43.5 \text{ kN/m}$

maximum bending moment  $(M_u) = \frac{w_u l^2}{8} = \frac{43.5 \times 4^2}{8} = 87 \text{ kNm}$

Now,

for economy, design balanced section,  $x_u = x_{u,max} = 0.46d = 0.46 \times 400 = 184 \text{ mm}$

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

$$\text{or, } 87 \times 10^6 = \frac{0.36 \times 25 \times d}{2} \times 0.46d (d - 0.42 \times 0.46d)$$

on solving we get,

$$d = 373.47 \approx 400 \text{ mm}$$

$$\text{so, } d = 400 \text{ mm} \Rightarrow b = \frac{d}{2} = 200 \text{ mm}$$

provide 200mm x 400mm rectangular beam.

again,  $M_u = 0.87 \frac{f_y A_{st}}{f_{ck} b} (d - \frac{f_y A_{st}}{f_{ck} b})$

$$\text{or, } 87 \times 10^6 = \frac{0.87 \times 500 \times A_{st}}{25 \times 200} (400 - \frac{500 \times A_{st}}{25 \times 200})$$

on solving we get,

$$A_{st} = 558.756 \text{ mm}^2$$

provide 3-16mm rebar

$$A_{st, \text{ provided}} = 3 \times 201 = 603 \text{ mm}^2 > 585.726 \text{ mm}^2$$

minimum  $A_{st}$  provided,  $= 0.85 \frac{b d}{f_y}$

$$= 0.85 \times \frac{200 \times 400}{500}$$

$$= 136 \text{ mm}^2 \text{ (min } A_{st} \text{ to be provided)}$$

$$\text{max}^m A_{st} \text{ provided} = 0.04 b D = 0.04 \times 200 \times 400$$

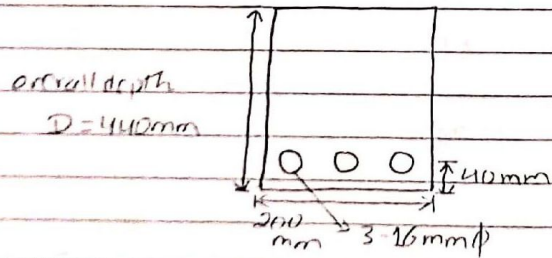
$$= 3200 \text{ mm}^2$$

$$D = d + 40 = 440 \text{ mm}$$

$$x_{u, \text{ max}} = 0.46 d = 0.46 \times 400 = 184 \text{ mm}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 500 \times 603}{0.36 \times 25 \times 200} = 146.725 \text{ mm}$$

$$146.725 < 184 \text{ mm} \quad \text{OK}$$



Note:

IF  $x_u \leq x_{u, \text{ max}}$  = 'balanced'

$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$  < 'under-reinforced'

IF  $x_u > x_{u, \text{ max}}$  over-reinforced section

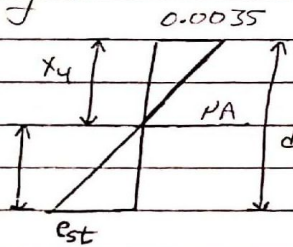
stress in steel  $f_{st} \neq 0.87 f_y$  because steel doesn't yield

$$x_u = \frac{P_{GL} A_{st}}{0.36 f_{ck} b}$$

$f_{st}$  can be found from code corresponding to strain in steel,  $\epsilon_{st} =$

$$0.0035 \frac{(d - x_u)}{x_u}$$

$$\therefore \epsilon_{st} = 0.0035 \left( \frac{d - x_u}{x_u} \right)$$



$$\epsilon_{st} = \frac{d - x_u}{x_u} \times 0.0035$$

$$\therefore \epsilon_{st} = 0.0035 \left( \frac{d - x_u}{x_u} \right)$$

code pg. 36 Table A

4) Determine the neutral axis depth and M<sub>02</sub> of beam section as shown. use M20 mix and Fe415 steel

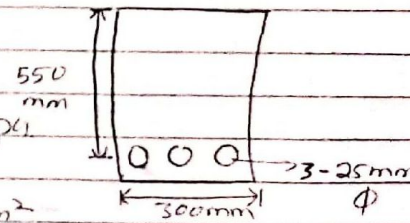
sol<sup>n</sup>

Here,

$$b = 300 \text{ mm}, d = 550 \text{ mm}, f_{ck} = 20 \text{ MPa}$$

$$f_y = 415 \text{ MPa}$$

$$A_{st} = 4 \times 1 \times 25^2 = 1963.495 \text{ mm}^2$$



$$x_{u, \text{ max}} = 0.45 d \text{ for Fe415}$$

$$= 0.45 \times 550 = 247.5 \text{ mm}$$

$$\text{Assuming } x_u \leq x_{u, \text{ max}}, x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 1963.495}{0.36 \times 20 \times 300} = 370.704 > 247.5 \text{ mm}$$

$$0.36 \times 20 \times 300$$

$$= 370.704 > 247.5 \text{ mm}$$



Now,  $x_u - f_{st} A_{sc} = 1963.495 \times f_{st}$   
 $0.36 f_{cr, b} \quad 0.36 \times 20 \times 300$

$x_u = 0.909 f_{st} \quad \text{--- (i)}$

First trial,

assume  $x_u = \frac{264 + 328}{2} = 296 \text{ mm}$

strain in steel,  $\epsilon_{st} = 0.0035 \left( \frac{d}{x_u} - 1 \right) = 0.0035 \left( \frac{550}{296} - 1 \right)$   
 $= 0.003$

stress in steel,  $f_{st} = 351.8 + \frac{(360.9 - 351.8)(300 - 276)}{380 - 276}$   
 $= 353.9$

strain ( $\epsilon_{st}$ )	stress ( $f_{st}$ )	code pg 36 table A
0.00276 $x_1$	351.8 $y_1$	at 415 N/mm <sup>2</sup>
0.003 $x$	? $y$	$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$
0.00380 $x_2$	360.9 $y_2$	$y - 351.8 = \frac{(360.9 - 351.8)}{(0.00380 - 0.00276)} \times (0.003 - 0.00276)$

$x_u = 0.909 \times 353.9 = 321.695 \text{ mm}$

Second trial,

$x_u = \frac{296 + 321}{2} = 308.5 \text{ mm}$

$\epsilon_{st} = 0.0035 \left( \frac{550}{308.5} - 1 \right) = 0.00273$   
 $\approx 0.00270$

strain ( $\epsilon_{st}$ )	stress ( $f_{st}$ )	code pg 36 table A
0.00270 $x_1$	342.8 $y_1$	
0.00273 $x$	? $y$	$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$
0.00276 $x_2$	351.8 $y_2$	$y - 342.8 = \frac{(351.8 - 342.8)}{(0.00276 - 0.00270)} \times (0.00273 - 0.00270)$

$y - 342.8 = \frac{(351.8 - 342.8)}{(0.00276 - 0.00270)} \times (0.00273 - 0.00270)$   
 $\times 10^5$

From linear interpolation

270 use 21-01 07

$f_{st} = 342.8 + \frac{351.8 - 342.8}{(276 - 241)} (273 - 241) = 350.257 \text{ MPa}$

$x_u = 0.909 \times 350.275 = 318.385 \text{ mm}$

Third trial,

$x_u = \frac{308 + 318}{2} = 313 \text{ mm}$

$\epsilon_{st} = 0.0035 \left( \frac{550}{313} - 1 \right) = 0.00265$   
 ↓ convert in stress & apply

$f_{st} = 342.8 + \frac{(351.8 - 342.8)(265 - 241)}{(276 - 241)} = 348.97 \text{ MPa}$

$x_u = 0.909 \times 348.97 = 317.214 \text{ mm}$

so,

fourth trial

neutral axis depth  $x_u = \frac{313 + 317}{2} = 315$

$\epsilon_{st} = 0.0035 \left( \frac{550}{315} - 1 \right) = 0.0026$

$f_{st} = 342.8 + \frac{351.8 - 342.8}{(276 - 241)} (260 - 241) = 347.685$

$x_u = 0.909 \times 347.685 = 316.045 \text{ mm}$

so, neutral axis depth ( $x_u$ ) = 315 mm

$$MOR = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

$$= 0.36 \times 20 \times 300 \times 315 \times (550 - 0.42 \times 315) \times 10^{-6} \text{ kNm}$$

$$= 284.203 \text{ kNm}$$

Intam of steel,  $MOR = f_{st} \cdot A_{st} (d - 0.42 x_u)$

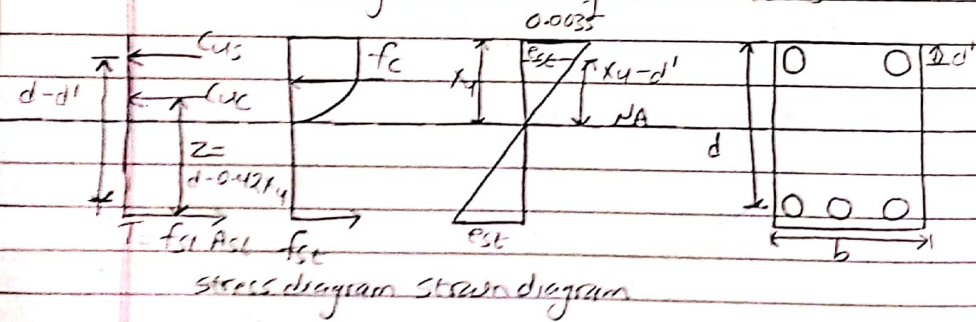
$$= 347.655 \times 1963.499 (550 - 0.42 \times 315) \times 10^{-6}$$

$$= 285.154 \text{ kNm}$$

Doubly reinforced beam

In doubly reinforced section, reinforcement is provided both in tension and compression zone. Doubly reinforced section is required when

- 1) size is restricted by architectural or aesthetic point of view.
- 2) There is application of reversal of load.
- 3) In continuous monolithic cast of beam and slab, the section is designed as doubly reinforced section.



$d'$  = effective cover to compression steel

$\epsilon_{sc}$  = strain in compression steel.

$$= 0.0035 (x_u - d') = 0.0035 \left(1 - \frac{d'}{x_u}\right)$$

$f_{st}$  = stress in tension steel.

$C_{uc}$  = compressive force by concrete  $= 0.36 f_{ck} b x_u$

$C_{us}$  = compressive force by compression steel  $= (f_{sc} - f_{cc}) A_{sc}$   
 $= (f_{sc} - 0.446 f_{ck}) \cdot A_{sc}$

$f_{sc}$  = stress in compression steel.

$A_{sc}$  = area of compression steel.

$T$  = Tension force by steel  $= f_{st} A_{st}$ .

at equilibrium

Tension force = compressive force.

$$T = C_{uc} + C_{us}$$

$$\text{or } f_{st} A_{st} = 0.36 f_{ck} b x_u + (f_{sc} - 0.446 f_{ck}) \cdot A_{sc}$$

$$\text{or } x_u = \frac{f_{st} A_{st} - (f_{sc} - 0.446 f_{ck}) A_{sc}}{0.36 f_{ck} b}$$

$$\therefore x_u = \frac{f_{st} A_{st} - (f_{sc} - 0.446 f_{ck}) A_{sc}}{0.36 f_{ck} b}$$

For Under-reinforced or balanced section i.e.  $x_u \leq x_{u, \max}$

$$f_{st} = 0.87 f_y$$

$$x_u = \frac{0.87 f_y A_{st} - (f_{sc} - 0.446 f_{ck}) A_{sc}}{0.36 f_{ck} b}$$

$$\text{yield strain} = \frac{f_y}{1.15 f_s} \quad \text{for Fe 250}$$

$$= 0.002 + \frac{f_y}{1.15 f_s} \quad \text{for Fe 415 and Fe 500}$$

If yield strain  $\leq \epsilon_{sc}$   
Then, compression steel yields and  
 $f_{sc} = 0.87 f_y$

Moment of resistance,

$$MOR = (A_{sc} z + A_{st} (d - d')) (f_{sc} - 0.446 f_{ck})$$

$$= 0.36 f_{ck} b x_u (d - 0.42 x_u) + (f_{sc} - 0.446 f_{ck}) A_{sc} (d - d')$$

\* Determine the ultimate MOR of doubly reinforced beam as shown. Use M20 mix and Fe 250 steel.

Sol<sup>n</sup>

$$b = 300 \text{ mm}$$

$$d = 550 \text{ mm}$$

$$d' = 50 \text{ mm}$$

$$f_{ck} = 20 \text{ MPa}$$

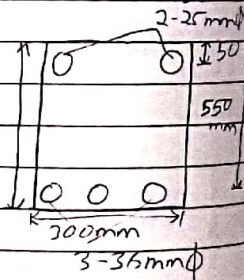
$$f_y = 250 \text{ MPa}$$

$$A_{st} = 3 \times \pi \times 36^2 = 3053.628 \text{ mm}^2$$

$$A_{sc} = 2 \times \pi \times 25^2 = 1982.777 \text{ mm}^2 = 981.747 \text{ mm}^2$$

$$x_{u, \text{max}} = 0.531 \text{ for Fe 250} = 0.531 \times 550 = 291.5 \text{ mm}$$

assuming  $x_u \leq x_{u, \text{max}}$  &  $f_{sc} = 0.87 f_y$



$$\text{neutral axis depth } x_u = \frac{f_{st} A_{st} (f_{sc} - 0.446 f_{ck}) A_{sc}}{0.36 f_{ck} b}$$

$$x_u = \frac{0.87 f_y A_{st} - (0.87 f_y - 0.446 f_{ck}) A_{sc}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 250 \times 3053.628 - (0.87 \times 250 - 0.446 \times 20) \times 1982.777}{0.36 \times 20 \times 300}$$

$$= 212.681 \text{ mm} < x_{u, \text{max}} = 291.5 \text{ mm}$$

So, our assumption of  $x_u \leq x_{u, \text{max}}$  i.e.  $f_{st} = 0.87 f_y$  is justified.

$$\text{strain in compression steel } \epsilon_{sc} = 0.0035 \left( 1 - \frac{d'}{x_u} \right)$$

$$= 0.0035 \left( 1 - \frac{50}{212.681} \right)$$

$$= 0.00267$$

$$\text{yield strain} = \frac{f_y}{1.15 f_s} \quad \text{for Fe 250}$$

$$= \frac{250}{1.15 \times 2 \times 10^5}$$

$$= 0.00108 < \epsilon_{sc} = 0.00267 \text{ OK}$$

So, our assumption  $f_{sc} = 0.87 f_y$  is also justified.

and calculated neutral axis depth is also correct.

$$\text{neutral axis depth } (x_u) = 212.681 \text{ mm}$$

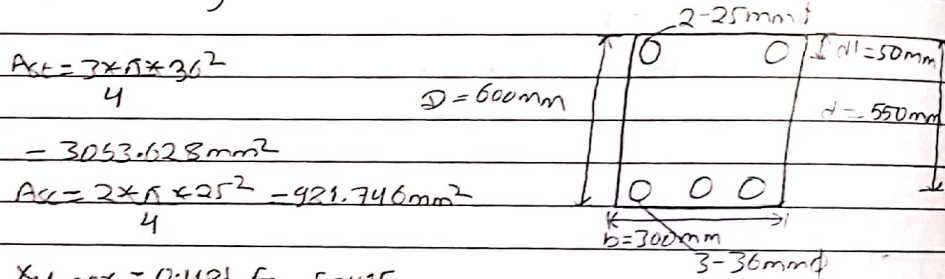
$$\text{moment of resistance } (MOR) = 0.36 f_{ck} b x_u \left( d - 0.42 x_u \right) + (f_{sc} - 0.446 f_{ck}) A_{sc} (d - d')$$

$$= 0.36 \times 20 \times 300 \times 212.681 \times (550 - 0.42 \times 212.681) + (0.87 \times 250 - 0.446 \times 20) \times 981.747 (550 - 50)$$

$$= 314.015856.5 \times 10^{-6}$$

$$= 314.015 \text{ kNm}$$

\* Determine the ultimate M<sub>OR</sub> of doubly reinforced beam as shown in figure. Use M20 mix and Fe415 steel.



$$A_{st} = \frac{3 \times \pi \times 36^2}{4}$$

$$= 3053.628 \text{ mm}^2$$

$$A_{sc} = \frac{2 \times \pi \times 25^2}{4} = 981.746 \text{ mm}^2$$

$$x_{u, \max} = 0.48d \text{ for Fe415}$$

$$= 0.48 \times 550 = 264 \text{ mm}$$

assuming  $x_u \leq x_{u, \max}$  and  $f_{sc} = 0.87 f_y$

$$M_u = f_{st} A_{st} - (f_{sc} - 0.446 f_{ck}) f_{sc} \times 0.36 f_{ck} b$$

$$= 0.87 \times 415 \times 3053.628 - (0.87 \times 215 - 0.446 \times 20) \times 981.746 \times 0.36 \times 20 \times 300$$

$$= 350.374 > x_{u, \max} = 264 \text{ mm}$$

∴ our assumption  $x_u \leq x_{u, \max}$  is wrong  
first trial,

$$x_u = \frac{264 + 350.374}{2} = 307.187 \text{ mm}$$

$$p_{st} = 0.0035 \left( \frac{d}{x_u} - 1 \right) = 0.0035 \left( \frac{550}{307.187} - 1 \right) = 0.00276$$

$$p_{sc} = 0.0035 \left( \frac{1 - d'}{x_u} \right) = 0.0035 \left( \frac{1 - 50}{307.187} \right) = 0.00293$$

$$f_{st} = 351.8$$

$$f_{sc} = 351.8 + \frac{360.4 - 351.8}{380 - 276} (243 - 276)$$

$$= 353.287 \text{ MPa}$$

$$M_u = f_{st} \times 3053.628 - f_{sc} \times 981.746 + 0.446 \times 20 \times 981.746 \times 0.36 \times 20 \times 300$$

$$= 1.413 f_{st} - 0.454 f_{sc} + 4.054$$

$$∴, M_u = 1.413 f_{st} - 0.454 f_{sc} + 4.054$$

$$∴, M_u = 1.413 \times 351.8 - 0.454 \times 353.287 + 4.054 = 340.755$$

second trial,

$$x_u = \frac{307 + 340}{2} = 323.5 \text{ mm}$$

$$p_{st} = 0.0035 \left( \frac{550}{323.5} - 1 \right) = 0.00245$$

$$p_{sc} = 0.0035 \left( \frac{1 - 50}{323.5} \right) = 0.00295$$

$$f_{st} = 342.8 + \frac{(351.8 - 342.8)}{(276 - 241)} (245 - 241)$$

$$= 343.828 \text{ MPa}$$

$$f_{sc} = 351.8 + \frac{(360.4 - 351.8)}{(380 - 276)} (245 - 276)$$

$$= 353.541 \text{ MPa}$$

$$x_u = 1.113 \times 348.828 - 0.1154 \times 353.541 + 4.054$$

$$= 329.775 \text{ mm}$$

Third trial,  $x_u = \frac{328 + 329}{2} = 326 \text{ mm}$

$$p_{st} = 0.0035 \left( \frac{550 - 1}{326} \right) = 0.00224$$

$$p_{sc} = 0.0035 \left( \frac{1 - 50}{328} \right) = 0.00296$$

$$f_{st} = 342.8 \text{ MPa}$$

$$f_{sc} = 351.8 + (360.9 - 351.8) \left( \frac{296 - 276}{380 - 276} \right)$$

$$= 353.55 \text{ MPa}$$

$$x_u = 1.113 \times 342.8 - 0.1154 \times 353.55 + 4.054 = 322.918$$

So, neutral axis depth ( $x_u$ ) =  $\frac{326 + 322.918}{2} = 326.95 \text{ mm}$

and,

Moment of Resistance

$$= 0.36 f_{ck} b x_u (d - 0.42 x_u) + (f_{sc} - 0.446 f_{ck}) A_{sc} (d - d')$$

$$= 0.36 \times 20 \times 300 \times 326.95 (550 - 0.42 \times 326.95) + (353.55 - 0.446 \times 20) \times 981.746 \times (550 - 50)$$

$$= 460.6098364 \times 10^6$$

$$= 460.61 \text{ kNm}$$

\* Determine the MUR of given section. use M20 mix and Fe415 steel.

Sol<sup>n</sup>

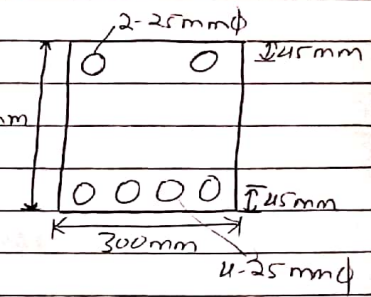
$$b = 300 \text{ mm}, \quad d = 700 - 45 = 655 \text{ mm}$$

$$d' = 45 \text{ mm}$$

$$f_{ck} = 20 \text{ MPa}, \quad f_y = 415 \text{ MPa}$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 25^2 = 1963.495 \text{ mm}^2$$

$$A_{sc} = 2 \times \frac{\pi}{4} \times 25^2 = 981.747 \text{ mm}^2$$



$$x_{u,max} = 0.48d$$

assuming  $x_u \leq x_{u,max}$  &  $f_{sc} = 0.87 f_y$

$$= 0.48 \times 655$$

$$= 314.4 \text{ mm}$$

$$x_u = \frac{p_{st} A_{st} - (f_{sc} - 0.446 f_{ck}) A_{sc}}{0.36 f_{ck} b}$$

$$= \frac{0.22 \times 415 \times 1963.495 - (0.87 \times 415 - 0.446 \times 20) \times 981.747}{0.36 \times 20 \times 300}$$

$$= 168.156 \text{ mm} < x_{u,max} = 314.4 \text{ mm}$$

So, our assumption of  $f_{ct} = 0.87 f_y$  is justified.

strain in compression steel =  $p_{sc} = 0.0035 \left( \frac{1 - d'}{x_u} \right)$

$$= 0.0035 \left( \frac{1 - 45}{168.156} \right)$$

$$= 0.00256$$

Yield strain =  $0.002 + \frac{f_y}{1.65 f_s} = 0.002 + \frac{415}{1.65 \times 2 \times 10^5} = 0.0025$

So, our assumption of  $f_{sc} = 0.87 f_y$  is not justified.

and calculated value of neutral axis depth is not correct.

First trial,  $X_u = \frac{314 + 168}{2} = 241 \text{ mm}$

$$e_{sc} = 0.0035 \left( \frac{1 - d'}{X_u} \right) = 0.0035 \left( \frac{1 - 45}{241} \right) = 0.00234$$

$$f_{sc} = 351.8 + \frac{(360.4 - 351.8)(354 - 276)}{380 - 276}$$

$$= 352.25 \text{ MPa}$$

$$X_u = 0.87 \times 45 \times 0.63 \times 495 = \frac{(f_{sc} - 0.446 \times 20) \times 981.747}{0.36 \times 20 \times 300}$$

$$X_u = 332.257 - 0.454 f_{sc} \quad \text{--- (1)}$$

$$X_u = 332.257 - 0.454 \times 352.5 = 172.272$$

Second trial,

$$X_u = \frac{241 + 172}{2} = 206.5$$

$$e_{sc} = 0.0035 \left( \frac{1 - 45}{206.5} \right) = 0.00273$$

$$f_{sc} = 342.8 + \frac{351.8 - 342.8}{276 - 241} \times (273 - 241)$$

$$= 351.028 \text{ MPa}$$

$$X_u = 332.257 - 0.454 \times 351.028 = 172.890 \text{ mm}$$

3<sup>rd</sup> trial,

$$X_u = \frac{206 + 172}{2} = 189$$

$$e_{sc} = 0.0035 \left( \frac{1 - 45}{189} \right) = 0.00266$$

$$f_{sc} = 342.8 + \frac{(351.8 - 342.8) \times (266 - 241)}{276 - 241}$$

$$= 349.328 \text{ MPa}$$

$$X_u = 332.257 - 0.454 \times 349.328 = 173.707 \text{ mm}$$

So, neutral axis depth ( $X_u$ ) = 173.707 mm

and, Moment of Resistance

$$M_u R = 0.36 f_{ck} \cdot b \cdot X_u (d - 0.42 X_u) + (f_{sc} - 0.446 f_{ck}) A_{sc} (d - d')$$

$$= 0.36 \times 20 \times 300 \times 173.707 (355 - 0.42 \times 173.707)$$

$$+ (349.328 - 0.446 \times 20) \times 981.747 (355 - 45)$$

$$= 422185478.7 \times 10^{-6}$$

$$= 422.185 \text{ KNm Ans}$$

First trial,  $X_u = \frac{314 + 168}{2} = 241 \text{ mm}$

$e_{sc} = 0.0035 \left( \frac{1-d'}{X_u} \right) = 0.0035 \left( \frac{1-45}{241} \right) = 0.00234$   
 0.00236 → 351.8

$f_{sc} = \frac{342.8 + (351.8 - 342.8)(241 - 276)}{380 - 276}$   
 0.00380 → 351.8  
 0.00284 → 352.5

$= 352.25 \text{ MPa}$

$X_u = 0.87 \times u_{1c} \times \frac{A_{st}}{b \times d} = (f_{cc} - 0.416 \times 20) \times \frac{A_{sc}}{0.36 \times 20 \times 300}$

$X_u = 332.257 - 0.454 f_{sc} \rightarrow \text{---}$

$X_u = 332.257 - 0.454 \times 352.5 = 172.222$

Second trial,

$X_u = \frac{241 + 172}{2} = 206.5$

$e_{sc} = 0.0035 \left( \frac{1-45}{206.5} \right) = 0.00273$

$f_{sc} = \frac{342.8 + (351.8 - 342.8)(273 - 241)}{276 - 241}$   
 0.00241 → 342.8  
 0.00276 → 351.8

$= 351.028 \text{ MPa}$

$X_u = 332.257 - 0.454 \times 351.028 = 172.890 \text{ mm}$

3<sup>rd</sup> trial

$X_u = \frac{206 + 172}{2} = 189$

$e_{sc} = 0.0035 \left( \frac{1-45}{189} \right) = 0.00266$  . code page 36

0.00241 → 342.8

$f_{sc} = \frac{342.8 + (351.8 - 342.8)(266 - 241)}{276 - 241}$   
 0.00276 → 351.8  
 0.00266 → 349.22

$= 349.228 \text{ MPa}$

$X_u = 332.257 - 0.454 \times 349.228 = 173.707 \text{ mm}$

So, neutral axis depth ( $X_u$ ) = 173.707 mm

and, Moment of resistance

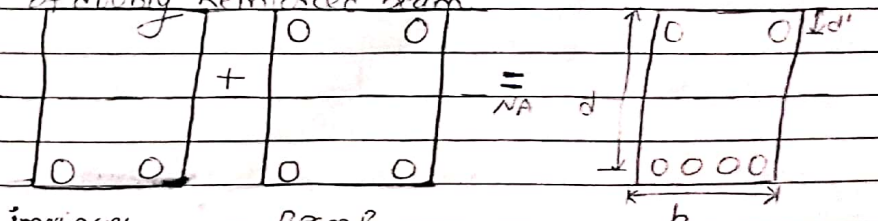
$M_u R = 0.36 f_{ck} \cdot b \cdot X_u (d - 0.42 X_u) + (f_{cc} - 0.416 f_{ck}) A_{sc} (d - d')$

$= 0.36 \times 20 \times 300 \times 173.707 (655 - 0.42 \times 173.707) + (349.228 - 0.416 \times 20) \times 981.747 (655 - 45)$

$= 422.18547297 \times 10^6$

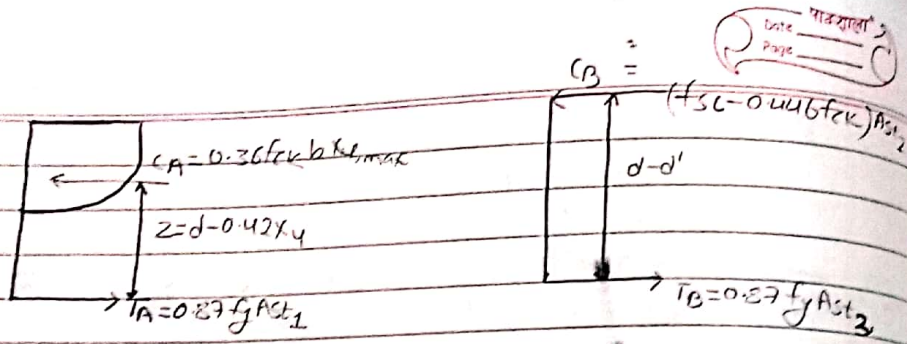
$= 422.185 \text{ KNm ANI}$

Design of doubly Reinforced beam



imaginary beam A

Beam B



1) calculate the moment of resistance of singly reinforced balanced section

$$M_{UL} = 0.36 f_{ck} b x_{u,max} (d - 0.42 x_{u,max})$$

2) if applied moment is greater than MOR of singly reinforced balanced section i.e.  $M_u > M_{UL}$ , then design doubly reinforced section is designed as

$$M_u - M_{UL} = 0.87 f_y A_{st2} (d - d') = (f_{sc} - 0.446 f_{ck}) A_{sc} (d - d')$$

3) calculate area of tension steel  $A_{st1}$  corresponding to singly reinforced balanced section

$$C_A = T_A = 0.36 f_{ck} b x_{u,max} = 0.87 f_y A_{st1}$$

4) calculate the area of tension steel  $A_{st2}$ , corresponding to additional moment.

$$A_{st2} = \frac{M_u - M_{UL}}{0.87 f_y (d - d')}$$

5) Total tension steel required,  $A_{st} = A_{st1} + A_{st2}$

6) calculate the area of compression steel

OR  $M = \frac{w u l^2}{4} = 22.3$  (given moment)  $\leftarrow$  given moment  $M_u > M_{UL}$  calculated  $M_{UL}$  of singly reinforced balanced section and size will not satisfy reinforced section design sit

$$A_{sc} = \frac{M_u - M_{UL}}{(f_{sc} - 0.446 f_{ck}) (d - d')} = \frac{0.87 f_y A_{st2}}{(f_{sc} - 0.446 f_{ck})}$$

8) Design a rectangular beam of effective length 6m. The superimposed load is 20 kN/m. Size of beam is restricted to 300mm x 700mm. OR use M20 mix and Fe415 steel.

Sol<sup>n</sup>

effective length ( $L_e$ ) = 6m

calculation of load

Superimposed load = 20 kN/m

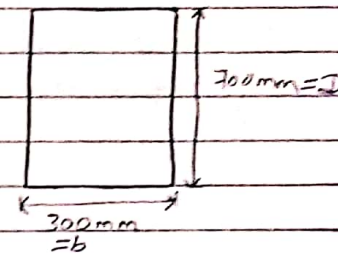
$\gamma_{f1} k_d =$  Self wt =  $0.25 \times 0.3 \times 0.7$  kN/m

$2.5$  kN/m<sup>3</sup> for RCC = 5.25 kN/m

Total load = 8.25 kN/m

(U.U) factored load =  $1.5 \times 8.25 = 12.375$  kN/m

factored moment ( $M_u$ ) =  $\frac{w u l^2}{8} = \frac{12.375 \times 6^2}{8} = 57.54375$  kNm



assume  $d' = 10\% \text{ of } D = 0.1 \times 700 = 70$  mm

effective depth ( $d$ ) =  $700 - 70 = 630$  mm

$x_{u,max} = 0.48 d = 0.48 \times 630 = 302.4$  mm for Fe415

MOR of singly reinforced balanced section,  $M_{UL} = 0.36 f_{ck} b x_{u,max} (d - 0.42 x_{u,max})$

$$= 0.36 \times 20 \times 300 \times 302.4 (630 - 0.42 \times 302.4) \times 10^{-6} \text{ kNm}$$

IMP = 328.546 kNm

Since,  $M_u > M_{UL}$ , doubly reinforced section has to be designed



area of tension steel corresponding to singly reinforced balanced section  
i.e.  $C_A = T_A$

$$A_{st1} = \frac{0.36 f_{ck} b X_{u,max}}{0.87 f_y} \quad 0.36 f_{ck} b X_{u,max} = 0.87 f_y A_{st1}$$

$$\therefore A_{st1} = \frac{0.36 f_{ck} b X_{u,max}}{0.87 f_y}$$

$$= \frac{0.36 \times 20 \times 300 \times 302.4}{0.87 \times 415}$$

$$= 1809.123 \text{ mm}^2$$

area of tension steel corresponding to additional moment

$$A_{st2} = \frac{M_u - M_{u1}}{0.87 f_y (d - d')}$$

$$= \frac{(575.437 - 328.546) \times 10^6}{0.87 \times 415 (630 - 70)} = 1221.096 \text{ mm}^2$$

Total tension steel,  $A_{st} = A_{st1} + A_{st2}$

$$= 1809.123 + 1221.096$$

$$= 3030.219 \text{ mm}^2$$

provide 5-28mm  $\phi$  rebar

$$A_{st, provided} = 5 \times 615 = 3075 \text{ mm}^2 > 3030.219 \text{ mm}^2$$

OK

Minimum  $A_{st}$  to be provided

$$A_{st, min} = 0.85 \frac{b d}{f_y}$$

$$= \frac{0.85 \times 300 \times 630}{415} = 387.108 < 3075 \text{ mm}^2$$

OK

max<sup>m</sup>  $A_{st}$  that can be provided,  $A_{st, max} = 0.04 b D$

$$= 0.04 \times 300 \times 700$$

$$= 8400 \text{ mm}^2 > 3075 \text{ mm}^2$$

OK

area of compression steel,  $A_{sc} = M_u - M_{u1}$   
 $(f_{sc} - 0.446 f_{ck})(d - d')$

$$= \frac{575.437 - 328.546}{(f_{sc} - 0.446 \times 20)(630 - 70)}$$

To find  $f_{sc}$

$$\frac{d'}{d} = \frac{70}{630} = 0.11 \quad \text{code page 38}$$

$$f_{sc} = 353 + (342 - 353) \times (0.11 - 0.1)$$

$$(0.15 - 0.1)$$

$$= 350.8 \text{ MPa}$$

fsc 415	0.10	0.15
	↓	↓
	353	342
	0.11 = 350.8	

now

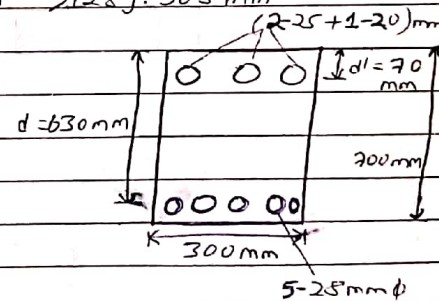
$$A_{sc} = \frac{(575.437 - 328.546) \times 10^6}{(350.8 - 0.446 \times 20)(630 - 70)}$$

$$= 1289.565 \text{ mm}^2$$

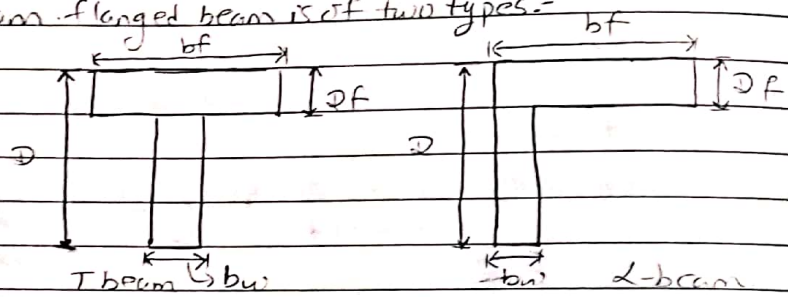
provide (2-25 + 1-20)mm  $\phi$  rebar

$$A_{sc, provided} = 2 \times 490 + 1 \times 314$$

$$= 1294 \text{ mm}^2 > 1289.565 \text{ mm}^2$$

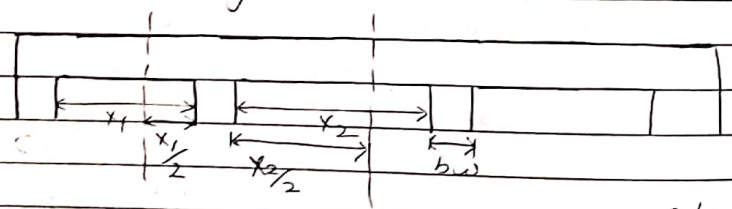


flanged beam - In most construction, beam and slab are cast monolithically. web portion called beam, bears tensile force where as flange portion slab bears compressive force. such combination of beam with web and flange is called flanged beam. flanged beam is of two types:-



bf = width of flange  
bw = width of web/rib  
Df = depth of flange  
D = overall depth.

effective width of flange, bf



$$bf = \frac{L_0}{6} + bw + 6Df$$

$$= bw + \frac{x_1}{2} + \frac{x_2}{2}$$

lesser of two values

code page 6  
CL 23.1.2

Types of problem

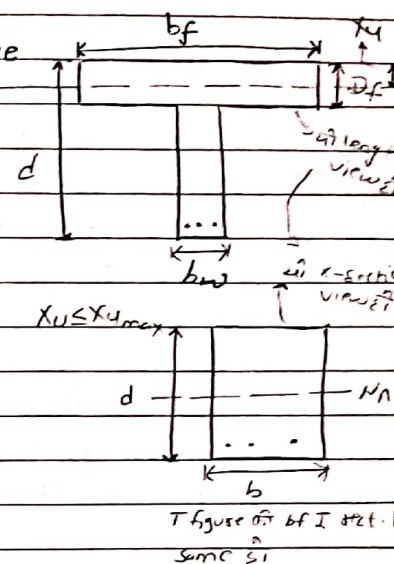
1) when neutral axis lies within flange

depth of neutral axis, ( $X_u \leq D_f$ ) NA

$$X_u = 0.87 f_y A_{st} / 0.36 f_{ck} b_f$$

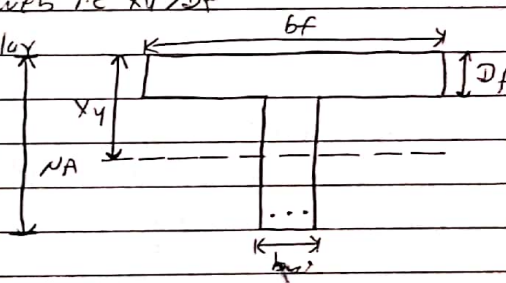
$$MOR = 0.36 f_{ck} b_f X_u (d - 0.42 X_u)$$

$$= 0.87 f_y A_{st} (d - \frac{f_y A_{st}}{f_{ck} b_f})$$



2) when neutral axis lies within web i.e.  $X_u > D_f$

Case 1) when depth of rectangular stress block diagram is greater than depth of flange i.e.  $3X_u > D_f \Rightarrow D_f < 0.428 X_u$



Total compressive force

(C) = compressive force by rectangular area ( $b_w \cdot X_u$ ) + compressive force by rectangular area ( $(bf - bw) \cdot D_f$ )

$$a_1 C = 0.36 f_{ck} b_w X_u + 0.446 f_{ck} (bf - bw) D_f$$

Total tensile force (T) =  $0.87 f_y A_{st}$

$X_u \leq X_{u,max}$

as  $C=T$

as  $0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) D_f = 0.87 f_y A_{st} C$

as  $x_u = \frac{0.87 f_y A_{st} - 0.446 f_{ck} (b_f - b_w) D_f}{0.36 f_{ck} b_w}$

$MOR = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.446 f_{ck} (b_f - b_w) D_f \times \left(\frac{d - D_f}{2}\right)$   $x_u > D_f$

$x_u > \frac{D_f}{0.428}$   
 $x_u \leq x_{u,max}$

Case II. when depth of rectangular stress block diagram is less than depth of flange i.e

$\frac{x_u}{D_f} < 0.428 \Rightarrow D_f > \frac{x_u}{0.428}$

In this case, depth of flange  $D_f$  is replaced by equivalent depth of flange ( $y_f$ ) where

$y_f = 0.15 x_u + 0.65 D_f$   $\nabla D_f$

now,  $C=T$

$0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) y_f$  (should not be greater than  $D_f$ )

$= 0.87 f_y A_{st}$   
 $\Rightarrow x_u = \frac{0.87 f_y A_{st} - 0.446 f_{ck} (b_f - b_w) y_f}{0.36 f_{ck} b_w}$

$MOR = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.446 f_{ck} (b_f - b_w) y_f \cdot (d - 0.5 y_f)$

97. A T beam of flange width  $b_f = 900\text{mm}$ , thickness  $100\text{mm}$  rib width  $b_w = 250\text{mm}$  has effective depth  $d = 525\text{mm}$  is reinforced with steel area of  $A_{st} = 4909\text{mm}^2$  find the ultimate moment of resistance of section use M20 and Fe250 steel.

Sol<sup>n</sup>

$b_f = 900\text{mm}$ ,  $D_f = 100\text{mm}$

$b_w = 250\text{mm}$ ,  $d = 525\text{mm}$

$A_{st} = 4909\text{mm}^2$

$x_{u,max} = 0.53 d$  for Fe250

$= 0.53 \times 525$

$= 278.25\text{mm}$

Assuming neutral axis lies within flange i.e  $x_u \leq D_f$

$\Rightarrow x_u \leq 100\text{mm}$

and  $x_u \leq x_{u,max} \Rightarrow x_u \leq 278.25\text{mm}$

$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f} = \frac{0.87 \times 250 \times 4909}{0.36 \times 20 \times 900} = 164.77\text{mm} < 100\text{mm}$  not OK

So our assumption is wrong.

Assume  $x_u \leq x_{u,max}$  and  $D_f \leq 0.428 \Rightarrow x_u > D_f$   
 $x_u \leq 278.25\text{mm}$

Assuming neutral axis lies within web, ( $x_u > D_f$ )

$\Rightarrow x_u > 100 - 233.645\text{mm}$   
 $0.428$

$x_u = \frac{0.87 f_y A_{st} - 0.446 f_{ck} (b_f - b_w) D_f}{0.36 f_{ck} b_w}$

$= \frac{0.87 \times 250 \times 4909 - 0.446 \times 20 (900 - 250) \times 100}{0.36 \times 20 \times 250}$

$= 271.059\text{mm} > 100\text{mm}$

$> 233.645\text{mm}$

$< 278.25\text{mm}$

$x_u > D_f$   
 $x_u > 233.645 \text{ i.e } > D_f$   
 $x_u \leq x_{u,max}$   
OK

so, neutral axis depth ( $x_u$ ) = 271.059 mm

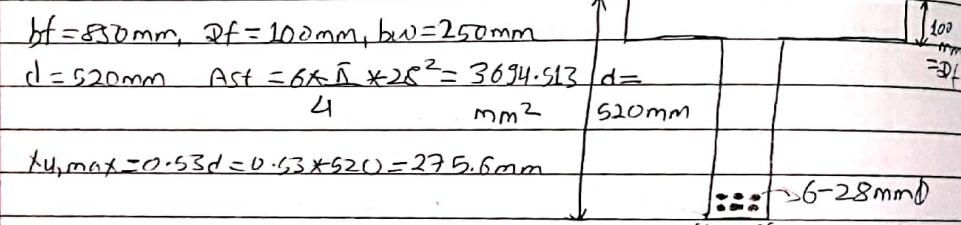
and Moment of resistance

$$= 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.446 f_{ck} (b_f - b_w) D_f (d - 0.5 D_f)$$

$$= 0.36 \times 20 \times 250 \times 271.059 (525 - 0.42 \times 271.059) + 0.446 \times 20 \times (900 - 250) \times 100 (525 - 0.5 \times 100)$$

$$= 476.01 \text{ kNm} \quad \times 10^6$$

10) Determine the ultimate moment of resistance of T-section as shown. Use M20 mix and Fe250 steel



assuming neutral axis lies within flange i.e.  $x_u \leq D_f \Rightarrow x_u \leq 100 \text{ mm}$  &  $x_u \leq x_{u,max}$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

$$= \frac{0.87 \times 250 \times 3694.513}{0.36 \times 20 \times 850}$$

$$= 131.30 \text{ mm} > 100 \text{ mm} = D_f$$

not OK

assuming neutral axis lies in web that is  $x_u > 100 \text{ mm}$  &  $\frac{D_f}{x_u} < 0.425 \Rightarrow x_u > 100 = 233.649 \text{ mm}$ ,  $x_u \leq x_{u,max}$

$$x_u = \frac{0.87 f_y A_{st} - 0.446 f_{ck} (b_f - b_w) D_f}{0.36 f_{ck} b_w}$$

$$= \frac{0.87 \times 250 \times 3694.513 - 0.446 \times 20 \times (850 - 250) \times 100}{0.36 \times 20 \times 250}$$

$$= 149.087 \text{ mm} < 233.645 \text{ mm} \quad (x_u > 233.645 \text{ mm})$$

not satisfied  
(not OK)

assuming neutral axis lies in web i.e.  $x_u > 100 \text{ mm}$  &  $\frac{D_f}{x_u} > 0.425$

$$\Rightarrow x_u < 100 \Rightarrow x_u < 233.645 \text{ mm} \text{ \& } x_u < x_{u,max} = 275.6 \text{ mm}$$

equivalent depth of flange ( $y_f$ ) =  $0.25 x_u + 0.65 D_f \neq D_f$

$$= 0.15 x_u + 0.65 \times 100$$

$$= 0.15 x_u + 65$$

C = T

$$0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 250 x_u + 0.446 \times 20 (850 - 250) (0.15 x_u + 65) = 0.87 \times 250 \times 3694.513$$

on solving we get

$$x_u = 175.071 \text{ mm} > 100 \text{ mm} \quad (x_u > D_f = 100 \text{ mm})$$

$$< 233.645 \text{ mm} \quad \left( \frac{D_f}{x_u} < \frac{0.425}{0.428} = 233.645 \text{ mm} \right)$$

$$\leq x_{u,max} = 275.6 \text{ mm} \quad (x_u \leq x_{u,max} = 275.6 \text{ mm})$$

so, neutral axis depth ( $x_u$ ) = 175.071 mm

$$y_f = 0.15 x_u + 65 \neq D_f$$

$$= 0.15 \times 175.071 + 65 \neq 100$$

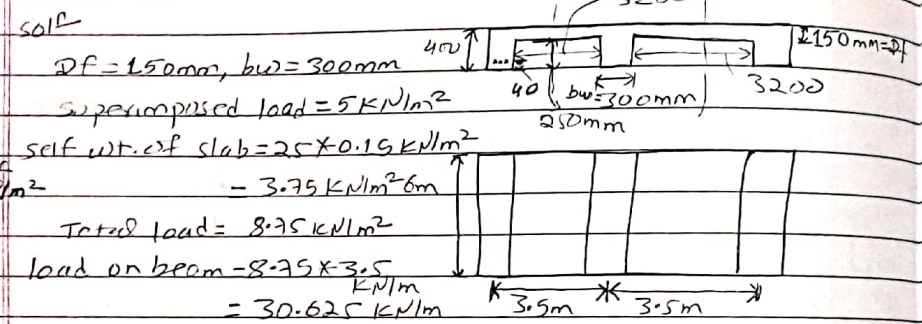
$$= 91.261 \text{ mm} \neq 100 \text{ mm} \text{ OK}$$

so, Moment of resistance =  $0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.446 f_{ck} (b_f - b_w) y_f (d - 0.5 y_f)$

$$= [0.36 \times 20 \times 250 \times 175.071 \times (520 - 0.42 \times 175.071) + 0.446 \times 20 \times (850 - 250) \times 91.261 \times (520 - 0.5 \times 91.261)] \times 10^6$$

$$= 372.391 \text{ kNm}$$

1) A T-beam floor consists of 15cm thick slab monolithic with 30cm wide beams. The beams are spaced at 3.5m (centre to centre) and effective span length is 6m. If superimposed load on slab is 5 kN/m<sup>2</sup>. Design an intermediate beam. Use M20 mix and Fe25 steel.



$\frac{Y \times DF}{kNm^2}$

$DF = 150mm, bw = 300mm$

superimposed load = 5 kN/m<sup>2</sup>

self wt. of slab = 25 × 0.15 kN/m<sup>2</sup>

= 3.75 kN/m<sup>2</sup> × 6m

Total load = 8.75 kN/m<sup>2</sup>

load on beam = 8.75 × 3.5

= 30.625 kN/m

assuming overall depth of beam 400mm with effective cover 40mm  
effective depth  $d = 400 - 40 = 360mm$

self wt. of portion of beam = 250 × 0.3 × 0.25 kN/m

$\frac{V_{concrete}}{kNm} = 1.875 kNm \rightarrow b_w \rightarrow (d - DF)$   
= 400 - 150 = 250mm

Total load = 32.5 kN/m (30.625 + 1.875)

Factored moment ( $M_u$ ) =  $w_u l^2 / 8 = 48.75 \times 6^2 / 8 = 219.375 kNm$

factored load ( $w_u$ ) = 32.5 × 1.5 = 48.75 kN/m

assuming neutral axis lies within flange  $x_u \leq 150mm$   
and  $x_u \leq x_{u,max}$

$M_u = 0.87 f_y A_{st} (d - \frac{f_y A_{st}}{f_{ck} b_f})$

$0.219.375 \times 10^6 = 0.87 \times 250 \times A_{st} (360 - \frac{250 \times A_{st}}{20 \times 2200})$

on solving we get.  $A_{st} = 2937.95mm^2$



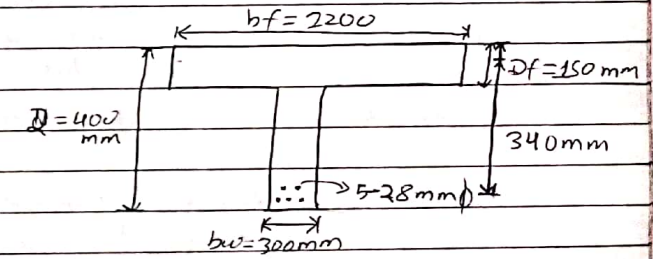
$= \frac{x_1 + x_2 + b_w}{2}$  breadth of flange  $b_f = d_o + b_w + 6DF$   
=  $\frac{3200 + 3200}{2} + 300 = 3500mm$   
=  $\frac{6000}{6} + 300 + 6 \times 150 = 2200mm$   
So small  $b_f = 2200mm$

provide 5-28mm  $\phi$  rebars,  $A_{st, provided} = 5 \times 615 = 3075mm^2$   
 $> 2937.95mm^2$   
OK

$x_u = 0.87 f_y A_{st} / 0.36 f_{ck} b_f = 0.87 \times 250 \times 3075 / 0.36 \times 20 \times 2200 = 42223mm < 150mm$   
 $< x_{u,max}$   
OK

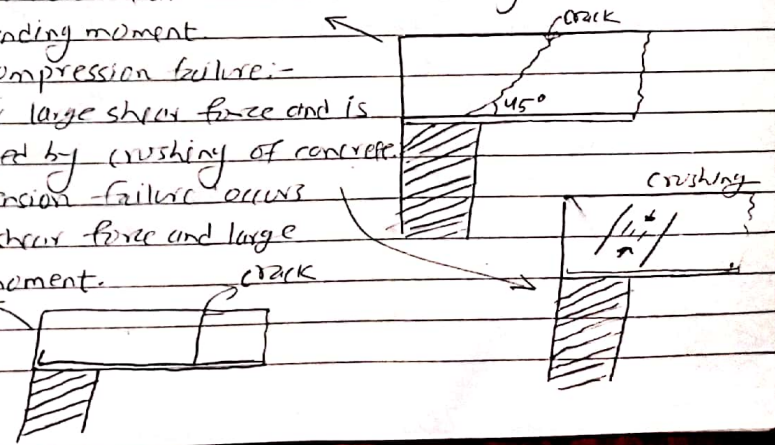
$A_{st, min} = 0.85 b_w d / f_y = 0.85 \times 300 \times 360 / 250 = 367.2mm^2$   
 $< 3075mm^2$   
OK

$A_{st, max} = 4\% \text{ of } b_w \times D = 0.04 \times 300 \times 400 = 4800mm^2 > 3075mm^2$   
OK



Design for shear.  
Types of shear failure.

- 1) Diagonal tension failure: - occurs under large shear force and less bending moment.
- 2) Diagonal compression failure: - occurs under large shear force and is characterised by crushing of concrete.
- 3) Flexural tension failure: - occurs under large shear force and large bending moment.



Shear strength of beam = shear strength of concrete + shear strength of shear reinforcement

$$V_u = V_{uc} + V_{us}$$

$V_u$  = factored shear force.

$V_{us}$  = shear force to be resisted by shear reinforcement

$V_{uc}$  = shear force resisted by concrete =  $T_c \cdot b \cdot d$

where

$T_c$  = design shear stress of concrete

$T_c$  depends upon % of steel and grade of concrete

(code page 40 - table 19)

shear reinforcement is needed when  $T_c < T_u < T_{c,max}$

$$T_u = \text{ultimate shear stress} = \frac{V_u}{b \cdot d}$$

$T_{c,max}$  = Maximum shear stress of concrete

(table 20, page 40)

shear force to be resisted by shear reinforcement,

$$V_{us} = V_u - V_{uc} \\ = V_u - T_c \cdot b \cdot d$$

\* Spacing of vertical stirrups or shear reinforcement

$S_u$  = spacing of vertical stirrups

$d$  = effective depth.

$A_{sv}$  = area of one vertical stirrups

shear force resisted by shear reinforcement,

$$V_{us} = \frac{f_y}{1.25} \times A_{sv} \cdot d$$

$$S_u \rightarrow A_{sv} \cdot d$$

$$\therefore \frac{A_{sv} \cdot d}{S_u}$$

code page 40

For inclined stirrups or a series of bars bent-up at different cross-section

$$V_{us} = 0.87 f_y A_{sv} \cdot d (\sin \alpha + \cos \alpha)$$

$S_u$

$\alpha$  = angle of inclined stirrups or bar to the axis of beam.

For single bar or single group of parallel bars, all bent-up at the same cross section.

$$V_{us} = 0.87 f_y A_{sv} \sin \alpha$$

Minimum area of shear reinforcement

$$A_o \geq \frac{0.4 b S_u}{0.87 f_y} \quad (\text{CL 26.5.1.6, page 18})$$

spacing,  $S_u \geq 100 \text{ mm}$

$$< 0.75 d$$

$$< 300 \text{ mm}$$

12) A RCC beam has effective depth of 500mm and breadth 300mm. It contains 4-25mm  $\phi$  bars. Calculate shear reinforcement needed for a factored shear force of 350kN for (i) M20 & Fe25

(ii) M25 and Fe415

Sol<sup>n</sup>

i) - factored shear force ( $V_u$ ) = 350kN,  $d = 500 \text{ mm}$ ,  $b = 300 \text{ mm}$

$$\% \text{ of steel} = \frac{A_{st} \times 100}{b \cdot d} = \frac{4 \times \frac{\pi}{4} \times 25^2}{300 \times 500} = 0.211 = 1.121\%$$

code page 40 for M20

(Case I) design shear stress of concrete ( $T_c$ ) =

$$0.62 + (0.67 - 0.62)(1.121 - 1) \quad 1.0 \rightarrow 0.62$$

$$(1.25 - 1) = 0.644 \text{ N/mm}^2 \quad 1.25 \rightarrow 0.67$$

ultimate shear stress,

$$1.121\% = 0.644 \text{ N/mm}^2$$

$$T_u = \frac{V_u}{b \cdot d} = \frac{350 \times 10^3}{300 \times 500} = 2.33 \text{ N/mm}^2$$

code pg 40 table 20  
 $\tau_{c, \text{max}}$  for M20 = 2.8 N/mm<sup>2</sup>

Since,  $\tau_c < \tau_u < \tau_{c, \text{max}}$ , shear reinforcement should be provided  
 shear force to be resisted by shear reinforcement

$$V_{us} = V_u - \tau_c b d$$

$$= 350 - 0.64 \times 350 \times 500 \times 10^{-3}$$

$$= 237.3 \text{ kN}$$

providing 2-legged 8mm  $\phi$  vertical stirrups,  $A_{sv} = \frac{2 \times \pi \times 8^2}{4}$

$$= 100.53 \text{ mm}^2$$

spacing

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87 \times 250 \times 100.53 \times 500}{237.3 \times 10^3}$$

$$= 46.07 \text{ mm} < 100 \text{ mm}$$

not OK

providing 100mm spacing

$$A_{sv} = \frac{V_{us} S_v}{0.87 f_y d} = \frac{237.3 \times 10^3 \times 100}{0.87 \times 250 \times 500} = 218.206 \text{ mm}^2$$

$$\text{or } \frac{2 \times \pi \times \phi^2}{4} = 218.206$$

$$\therefore \phi = \sqrt{\frac{2 \times 218.206 \times 4}{\pi}} = 11.786 \approx 12 \text{ mm}$$

minimum area of shear reinforcement,  $A_0 = \frac{0.4 b S_v}{0.87 f_y}$

So, provide 2-legged 12mm  $\phi$  vertical stirrups at spacing 100mm c/c.

$$= \frac{0.4 \times 250 \times 100}{0.87 \times 250} = 64.36 \text{ mm}^2 < 226.17 \text{ mm}^2$$

↓ OK  
 $\frac{2 \times \pi \times 12^2}{4}$   
 4

ii) factored shear force ( $V_u$ ) = 350 kN,  $d = 500 \text{ mm}$ ,  $b = 350 \text{ mm}$   
 $\therefore$  % of steel =  $\frac{A_{st} \times 100}{b d} = \frac{4 \times \pi \times 25^2}{350 \times 500} = 0.011 = 1.125\%$

Design shear stress of concrete ( $\tau_c$ )

1.00  $\rightarrow$  0.64  
 1.125  $\rightarrow$  ?  
 1.25  $\rightarrow$  0.70     1.125  $\hat{y} = 0.669 \text{ N/mm}^2$

From interpolation

$$\tau_c = 0.669 \text{ N/mm}^2$$

ultimate shear stress

$$\tau_u = V_u = 350 \times 10^3 = 2 \text{ N/mm}^2$$

$\tau_{c, \text{max}}$  for M20 = 2.8 N/mm<sup>2</sup> code pg 40 table 20

Since,  $\tau_c < \tau_u < \tau_{c, \text{max}}$ , shear reinforcement should be provided. Shear force to be resisted by shear reinforcement.

$$V_{us} = V_u - \tau_c b d$$

$$= 350 - 0.669 \times 350 \times 500 \times 10^{-3}$$

$$= 232.925 \text{ kN}$$

providing 2-legged 8mm  $\phi$  vertical stirrups,  $A_{sv} = \frac{2 \times \pi \times 8^2}{4} = 100.53 \text{ mm}^2$

spacing

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87 \times 250 \times 100.53 \times 500}{232.925 \times 10^3} = 93.99 < 100 \text{ mm}$$

AS  $S_v \geq 100 \text{ mm}$  is not satisfied, so not OK

providing 100mm spacing

$$A_{sv} = \frac{V_{us} S_v}{0.87 f_y d} = \frac{232.925 \times 10^3 \times 100}{0.87 \times 250 \times 500} = 129.026 \text{ mm}^2$$

$$\text{or } \frac{2 \times \pi \times \phi^2}{4} = 129.026 \quad \therefore \phi = \sqrt{\frac{129.026 \times 4}{2 \times \pi}} = 9.06 \approx 10 \text{ mm}$$

minimum area of shear reinforcement  $A_0 = \frac{0.4 b S_v}{0.87 f_y} = \frac{0.4 \times 350 \times 100}{0.87 \times 250} = 35.775 \text{ mm}^2$

$$\frac{2 \times \pi \times 10^2}{4} = 157.079 \text{ mm}^2 > 35.775 \text{ mm}^2$$

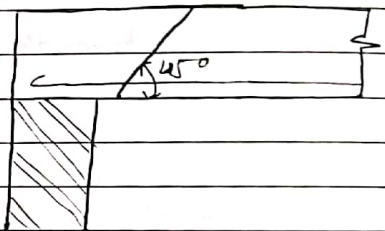
So, provide 2-legged 10mm $\phi$  vertical stirrups at spacing of 100mm c/c.

2015 Spring (5)

\* Explain in brief the type of shear failure in beam with neat sketches

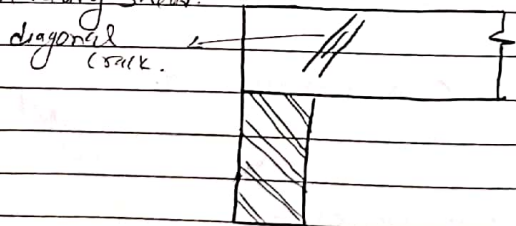
• Diagonal Tension failure.

If a section is under large shear force and small bending moment a diagonal tension crack is observed. Such cracks are normally at  $45^\circ$  with the horizontal.



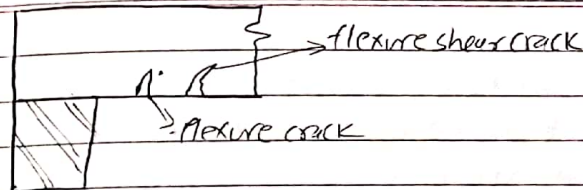
• Diagonal compression failure.

If the section is under large shear force and fails by forming a compression crack. It is indicated by crushing or concrete at support and spalling. Normally, it occurs in beam which are reinforced against heavy shear.



• Flexure shear failure

If the section is under large bending moment but small shear force, the section fails by forming a large vertical crack at around mid-span at  $90^\circ$ .



Mathematically  $\tan 2\alpha = \frac{2E}{\sigma_b}$

at support.

$\sigma_b \geq 0$

$\therefore \tan 2\alpha = 2 \times \infty = \infty$

or,  $2\alpha = \tan^{-1}(\infty) = \frac{\pi}{2}$

$\therefore \alpha = \frac{\pi}{4} = 45^\circ$

at mid-span:  $\tau \approx 0$

$\tan 2\alpha = \frac{2 \times 0}{\sigma_b} = 0$

or,  $2\alpha = \tan^{-1}(0) = 0$

$\therefore \alpha = 0^\circ$  (with vertical)

$\therefore \alpha = 90^\circ$  (with horizontal)

2016 fall 2a) Explain about the partial safety factor & factor of safety. Derive the necessary expression for the development length (both anchorage & flexural bond).

1st part see 2.6

2nd part.

a) Anchorage bond  
by flexural bond

$\therefore \tau_b d = \frac{V}{\pi \phi \alpha}$

This equation gives the flexural bond stress in the tension reinforcement at any section of the beam. If there are bars of different sizes then  $\tau_b d = \frac{V}{\pi \phi \alpha \cdot N}$



where  $N$  is no. of bars

From Anchorage bond, we have

$$0.87 f_y A_{st} = \text{Dev. } \pi \phi L_d N$$

using above equation we get.

$$0.87 f_y A_{st} = \frac{V}{\phi} \times \pi \phi L_d N$$

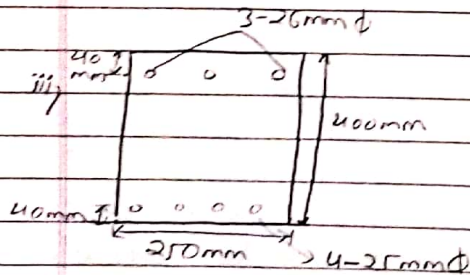
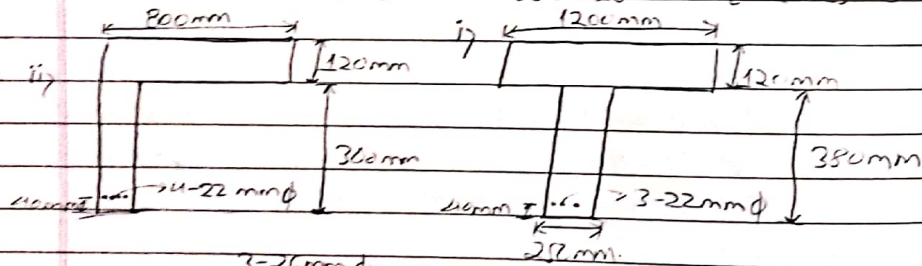
$$\therefore 0.87 f_y A_{st} = \frac{V}{\phi}$$

$$\therefore L_d = \frac{0.87 f_y A_{st} \phi}{V} = \frac{M}{V}$$

$L_d$  is further increased by 30% for compression.

$$\text{Then, } L_d = 1.3 \frac{M}{V} + \phi$$

13. A beam shown in fig is subjected to factored shear force of 200 kN. calculate shear reinforcement use M20 mix and Fe25 steel



i) sof

$$d = D + 380 - 40 = 460 \text{ mm}$$

$$\text{1.07 steel} = \frac{A_{st} \times 100}{b \times d} = \frac{3 \times \pi \times 22^2}{4 \times 100} \times 100 = 0.991 \approx 1.1$$

$$\frac{250 \times 460}{\text{for } 1.2 \times 20 \text{ } \tau_c = 0.6 \text{ N/mm}^2}$$

design shear stress of concrete ( $\tau_c$ ) = 0.62 N/mm<sup>2</sup> (code page)

$$\tau_{c, \text{max}} = 2.8 \text{ N/mm}^2$$

$$\text{ultimate shear stress } (\tau_u) = \frac{V_u}{b \times d} = \frac{200 \times 10^3}{250 \times 460} = 1.739 \text{ N/mm}^2$$

$\tau_c < \tau_u < \tau_{c, \text{max}}$  so, shear reinforcement is required

shear force to be resisted by steel reinforcement,  $V_{us} = V_u - \tau_c \cdot b \cdot d$

$$V_u = V_u + V_{us} \Rightarrow V_{us} = V_u - \tau_c \cdot b \cdot d = 200 - 0.62 \times 250 \times 460 \times 10^{-3}$$

$$\therefore V_{us} = 128.7 \text{ kN}$$

providing 8mm  $\phi$  2-legged vertical stirrups,  $A_{sv} = 2 \times \pi \times 8^2$

$$= 100.53 \text{ mm}^2$$

$$\text{spacing, } s_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87 \times 215 \times 100.53 \times 460}{128.7 \times 10^3}$$

$$= 129.73 \text{ mm} \approx 128 \text{ mm}$$

$$\geq 100 \text{ mm}$$

$$< 300 \text{ mm}$$

$$< 0.75d = 0.75 \times 460 = 345 \text{ mm}$$

minimum area of shear reinforcement,  $A_s = 0.4 b \cdot s_v$

$$= \frac{0.4 \times 250 \times 128}{0.87 \times 215}$$

$$= 35.45 \text{ mm}^2 < 100.53 \text{ mm}^2$$

So, provide 8mm  $\phi$  2-legged vertical stirrups @ 100mm c/c

ii) Sol<sup>n</sup>

$$d = 400 - 40 = 360 \text{ mm}$$

$$\% \text{ of steel} = \frac{A_{st} \times 100}{b \cdot d} = \frac{4 \times \frac{\pi}{4} \times 22^2}{250 \times 360} = 2.181\%$$

$$\text{design shear stress of concrete } (T_c) = \frac{0.29 + (0.81 - 0.79)}{(2.25 - 2)} \times (2.181 - 2) = 0.204 \text{ N/mm}^2$$

$$\text{Ultimate shear stress } (T_u) = \frac{V_u}{b \cdot d} = \frac{200 \times 10^3}{250 \times 360} = 2.22 \text{ N/mm}^2$$

$T_c < T_u < T_{c, \text{max}}$  So, shear reinforcement is required.  
 $0.804 \text{ N/mm}^2 < 2.22 \text{ N/mm}^2 < 2.28 \text{ N/mm}^2$

Shear force to be resisted by shear reinforcement,

$$V_{us} = V_u - T_c \cdot b \cdot d = 200 - 0.804 \times 250 \times 360 \times 10^{-3} = 127.64 \text{ kN}$$

providing 8mm  $\phi$  2-legged vertical stirrups,  $A_{sv} = \frac{2 \times \pi \times 8^2}{4} = 100.53 \text{ mm}^2$

$$\text{spacing, } s_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

$$= \frac{0.87 \times 415 \times 100.53 \times 360}{127.64 \times 10^3}$$

$$= 102.37 \text{ mm} \approx 100 \text{ mm}$$

$$\geq 100 \text{ mm}$$

$$< 300 \text{ mm}$$

$$< 0.75d = 0.75 \times 360 = 270 \text{ mm}$$

$$\text{minimum area of shear reinforcement, } A_{sv} = \frac{0.4 b_s V_u}{0.87 f_y}$$

$$= \frac{0.4 \times 250 \times 10^3}{0.87 \times 415}$$

$$= 27.696 \text{ mm}^2 < 100.53 \text{ mm}^2$$

So, provide 8mm  $\phi$  2-legged vertical stirrups @ 100mm c/c.

ii) Sol<sup>n</sup>

$$d = 120 + 300 - 40 = 380 \text{ mm}$$

$$\% \text{ of steel} = \frac{A_{st} \times 100}{b \cdot d} = \frac{4 \times \frac{\pi}{4} \times 22^2}{300 \times 380} = 1.33\%$$

design shear stress of concrete  $(T_c) =$

$$1.25 \rightarrow 0.62$$

$$1.50 \rightarrow 0.72$$

$$1.33\% = 0.652 \text{ N/mm}^2$$

$$\text{ultimate shear stress } (T_u) = \frac{V_u}{b \cdot d} = \frac{200 \times 10^3}{300 \times 380} = 1.754 \text{ N/mm}^2$$

$T_c < T_u < T_{c, \text{max}}$  so shear reinforcement is required  
 $0.652 \text{ N/mm}^2 < 1.754 \text{ N/mm}^2 < 2.28 \text{ N/mm}^2$

shear force to be resisted by shear reinforcement,

$$V_{us} = V_u - T_c \cdot b \cdot d = 200 - 0.652 \times 300 \times 380 \times 10^{-3} = 125.672 \text{ kN}$$

providing 8mm  $\phi$  2-legged vertical stirrups

$$A_{sv} = \frac{2 \times \pi \times 8^2}{4} = 100.53 \text{ mm}^2$$

$$= 100.53 \text{ mm}^2$$

$$\text{spacing } s_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87 \times 415 \times 100.53 \times 380}{125.672 \times 10^3} = 109.75$$

$$\geq 100 \text{ mm}$$

$$\geq 100 \text{ mm}$$

$$= 100 \text{ mm}$$

so provide 8mm  $\phi$  2-legged vertical stirrups at 100mm c/c

Design of beam for torsion:

Design for shear:

equivalent shear force,  $V_e = V_u + 1.6 \frac{T_u}{b}$

where

$V_u$  = ultimate shear force

(CL 41.3.1 page 42)

$T_u$  = torsional moment

$b$  = breadth of beam

Design for bending moment

equivalent torsional moment,  $M_t = \frac{T_u (1 + D/b)}{1.7}$

equivalent moment  $M_e = M_u + M_t$

if  $M_t > M_u$ ; longitudinal flexural reinforcement in compression zone should be provided with equivalent moment

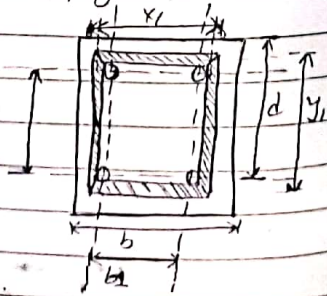
$M_e = M_u - M_t$ .  $M_e$  is assumed to act in opposite sense to that of  $M_u$ .

Note: if overall depth of beam exceeds 250mm i.e.  $D > 250$ mm, side face reinforcement should be provided with area of steel not less than 0.1% of  $bD$ . (CL 26.5.1.3, page 26)

Spacing of shear reinforcement,  $S_v$

$b_1$  = centre to centre distance between longitudinal bars in the dirn of breadth.

$d_1$  = centre to centre distance between longitudinal bars in the dirn of depth.



$x_1$  = shortest dimension of centre to centre distance between legs of stirrups

$y_1$  = longest dimension of centre to centre distance between legs of stirrups

$S_v = 0.87 f_y A_{sv} / d$   $V_u$  = factored shear force.

$\frac{T_u + V_u}{b_1 \cdot 2.5}$

$S_v / x_1 < \frac{(x_1 + y_1)}{4}$  (CL 41.4.3 pg 42)

$< 300$ mm (CL 26.5.1.3 page 18)  
 $\geq 100$ mm.

minimum area of shear reinforcement  $A_{sv} = \frac{(f_c - f_{ct}) \cdot b \cdot S_v}{0.87 f_y}$

14) Design a section of rectangular beam 500mm wide & 700mm deep subjected to a bending moment of 200 kNm, twisting moment of 15 kNm (torsional moment) & shear force of 150 kN at ultimate. Use M20 mix and Fe25 grade steel.

Soln

$M_u = 200$  kNm,  $T_u = 15$  kNm

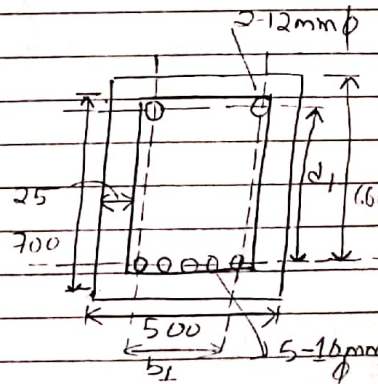
$V_u = 150$  kN

assuming 40mm effective cover, effective depth  $(d) = 700 - 40 = 660$  mm

Design for bending moment & equivalent torsional moment

$M_t = \frac{T_u (1 + D/b)}{1.7}$

$\frac{1.7}{1.7} = \frac{15 (1 + \frac{700}{500})}{1.7} = 21.176$  kNm



200 kNm 21.176 kNm

Since  $M_u > M_t$ , no need of longitudinal flexural reinforcement in compression zone.

equivalent bending moment,  $M_{e1} = M_u + M_t = 200 + 21.176 = 221.176 \text{ kNm}$

$x_u, \text{max} = 0.48d$  for Fe415  
 $= 0.48 \times 660 = 316.8 \text{ mm}$

MOM of singly reinforced balanced section,

$M_u = 0.36 f_c k b x_u, \text{max} (d - 0.42 x_u, \text{max})$   
 $= 0.36 \times 20 \times 500 \times 316.8 \times (660 - 0.42 \times 316.8) \times 10^{-6}$   
 $= 600.969 \text{ kNm}$

600.969 kNm 221.176 kNm

Since,  $M_u < M_{e1}$ , singly reinforced section is designed.

area of reinforcement for  $M_{e1}$

$M_{e1} = 0.87 f_y A_{st} (d - \frac{f_y A_{st}}{f_c k b})$

or,  $221.176 \times 10^6 = 0.87 \times 415 \times A_{st} (660 - \frac{415 A_{st}}{20 \times 500})$

on solving we get  
 $A_{st} = 989.766 \text{ mm}^2$

provide  $\phi$ -16mm rebars,  $A_{st}$  provided =  $5 \times 201 = 1005 \text{ mm}^2$   
 $> 989.766 \text{ mm}^2$

check for neutral axis depth,

$x_u = \frac{0.87 f_y A_{st}}{0.36 f_c k b} = \frac{0.87 \times 415 \times 1005}{0.36 \times 20 \times 500} = 100.793 \text{ mm}$

minimum area of steel  $(A_{st})_{\text{min}} = 0.85 \frac{b d}{f_y}$

$= \frac{0.85}{415} \times 500 \times 660 = 675.903 \text{ mm}^2$   
 $< 1005 \text{ mm}^2$

max area of steel  $(A_{st})_{\text{max}} = 4\% \text{ of } b d$

$= 0.04 \times 500 \times 700 = 14000 \text{ mm}^2 > 1005 \text{ mm}^2 \text{ OK}$

Design for shear

equivalent shear force  $(V_e) = V_u + 1.6 T_u = 150 + 1.6 \times 15 = 198 \text{ kN}$

ultimate shear stress  $(\tau_e) = \frac{V_e}{b d} = \frac{198 \times 10^3}{500 \times 660} = 0.6 \text{ N/mm}^2$

% of steel =  $\frac{A_{st} \times 100}{b d} = \frac{1005 \times 100}{500 \times 660} = 0.304\%$

Design shear stress of concrete,  $\tau_c = 0.36 + (0.48 - 0.36) \times (0.5 - 0.25) = 0.385 \text{ N/mm}^2$

$\tau_{u, \text{max}} = 2.8 \text{ N/mm}^2$

$\tau_c < \tau_e < \tau_{c, \text{max}}$ , so shear reinforcement is needed providing

2-legged 8mm  $\phi$  vertical stirrups,

$A_{sv} = 2 \times \pi \times r^2 = 100.53 \text{ mm}^2$

Assuming 2-legged 8mm  $\phi$  vertical stirrups,

$A_{sv} = 2 \times \pi \times r^2 = 100.53 \text{ mm}^2$

assuming 25mm clear cover on all side.

$b_1 = 500 - 25 - 25 - 8 - 10 - 16 = 418 \text{ mm}$

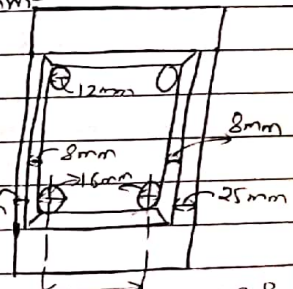
$d_1 = 660 - 25 - 8 - 12 = 621 \text{ mm}$

$x_1 = 500 - 25 - 25 - \frac{8}{2} - \frac{8}{2} = 442 \text{ mm}$

$y_1 = 700 - 25 - 25 - \frac{8}{2} - \frac{8}{2} = 642 \text{ mm}$

$S_v = 0.87 f_y A_{sv} d_1 = \frac{0.87 \times 415 \times 100.53 \times 621}{15 \times 10^6 + 150 \times 10^3} = 235.073 \text{ mm}$

$\frac{T_u}{b_1} + \frac{V_u}{2.5} = \frac{150}{418} + \frac{150}{2.5} = 230 \text{ mm}$



$$S_v = 230 \text{ mm} < x_1 = 1142 \text{ mm}$$

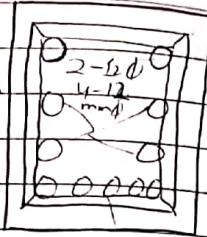
$$\leq \left( \frac{x_1 + y_1}{4} \right) = \left( \frac{1142 + 642}{4} \right) = 271 \text{ mm}$$

< 300 mm OK

minimum area of shear reinforcement

$$A_{sv} = \frac{(f_c - f_{cr}) b_s v}{0.87 f_y} = \frac{(0.6 - 0.385) \times 500 \times 230}{0.87 \times 415} = 68.48 \text{ mm}^2 < 100.53 \text{ mm}^2 \text{ OK}$$

So, provide 8mm $\phi$  2-legged closed vertical stirrups @ 230mm c/c.



15) Design a torsional reinforcement in a rectangular beam section 350mm wide and 750mm deep subjected to ultimate twisting moment 140 kNm combined with bending moment 200 kNm and ultimate shear force 110 kN. Use M25 concrete and Fe 415 steel.

$$(2-12+1-10) \text{ mm} \phi$$

So<sup>n</sup>

$$M_u = 200 \text{ kNm}, V_u = 110 \text{ kN}$$

$$T_u = 140 \text{ kNm}$$

assuming 40mm effective cover,

$$\text{effective depth } (d) = 750 - 40 = 710 \text{ mm}$$

Design for bending moment.

$$\text{equivalent torsional moment, } M_t = T_u \left( 1 + \frac{d}{b} \right)$$

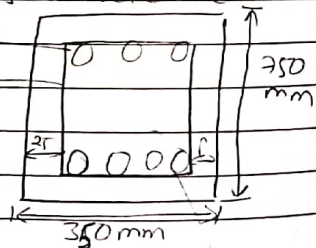
$$\frac{1.7}{1.7}$$

$$= 140 \left( 1 + \frac{750}{350} \right)$$

$$\frac{1.7}{1.7}$$

$$= 258.823 \text{ kNm}$$

$$M_u = 200 \text{ kNm}, V_u = 110 \text{ kN}, T_u = 140 \text{ kNm}$$



$$(2-28+2-25) \text{ mm}$$

Since  $M_t > M_u$ , flexural longitudinal reinforcement at compression zone should be provided

$$M_{e1} = M_u + M_t = 200 + 258.823 = 458.823 \text{ kNm} \text{ (at bottom)}$$

$$M_{e2} = M_t - M_u = 258.823 - 200 = 58.823 \text{ kNm} \text{ (at top)}$$

$$x_{u, \max} = 0.48 d = 0.48 \times 710 = 340.8 \text{ mm}$$

MOR for singly reinforced balanced section,

$$M_{uL} = 0.36 f_{ck} b x_{u, \max} (d - 0.42 x_{u, \max}) = 0.36 \times 25 \times 350 \times 340.8 \times (710 - 0.42 \times 340.8) \times 10^{-6}$$

$$= 608.539 \text{ kNm}$$

Since  $M_{uL} > M_{e1}$ , singly reinforced section is designed.

area of steel,  $A_{st1}$  for  $M_{e1}$

$$M_{e1} = 0.87 f_y A_{st1} (d - \frac{f_y A_{st1}}{2 f_{ck} b})$$

$$\text{or } 458.823 \times 10^6 = 0.87 \times 415 \times A_{st1} \left( 710 - \frac{415 \times A_{st1}}{25 \times 350} \right)$$

on solving we get,

$$A_{st1} = 2078.433 \text{ mm}^2$$

provide (2-28mm + 2-25mm)  $\phi$  rebars

$$A_{st} \text{ provided} = 2 \times 616 + 2 \times 490 = 2210 \text{ mm}^2$$

$$> 2078.433 \text{ mm}^2 \text{ OK}$$

$$\text{neutral axis depth, } x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 2210}{0.36 \times 25 \times 350} = 253.308 \text{ mm} < x_{u, \max} = 340.8 \text{ mm} \text{ OK}$$

$$\text{minimum area of steel } (A_{st})_{\min} = \frac{0.85 b d}{f_y} = \frac{0.85 \times 350 \times 710}{415} = 568.975 \text{ mm}^2 < 2210 \text{ mm}^2 \text{ OK}$$

$$\text{max}^m \text{ area of steel } (A_{st})_{\max} = 4\% \text{ of } b d = 0.04 \times 350 \times 750 = 10500 \text{ mm}^2 > 2210 \text{ mm}^2 \text{ OK}$$

As<sub>t2</sub> for MC2

$$M_{C2} = 0.87 f_y A_{st} (d - \frac{f_y A_{st} z}{f_{ck} b})$$

$$0.58823 \times 10^6 = 0.87 \times 415 \times A_{st2} (710 - \frac{415 \times A_{st2}}{25 \times 350})$$

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On solving,  $A_{st2} = 233.097 \text{ mm}^2$

provide (2-12+1-12)mm $\phi$  r bars, (As<sub>t</sub>)  
 $\text{provide } d = 2 \times 113 + 78 = 304 \text{ mm}^2$   
 $> 233.097 \text{ mm}^2$   
OK

design for shear

Equivalent shear

$$V_e = V_u + 1.6 T_u = 110 + \frac{1.6 \times 140}{0.35} = 750 \text{ kN}$$

ultimate shear stress ( $\tau_u$ ) =  $\frac{V_e}{b d} = \frac{750 \times 10^3}{350 \times 710} = 3.018 \text{ N/mm}^2$

% of steel =  $\frac{A_{st} L}{b d} \times 100 = \frac{2210}{350 \times 710} \times 100 = 0.883\%$  (see code pg 40)

design shear stress of concrete ( $\tau_c$ ) =  $\frac{0.57 + (0.60 - 0.57) \times 0.884}{1 - 0.75} = 0.608 \text{ N/mm}^2$

$T_{u, \max} = 3.1 \text{ N/mm}^2$  for MC2 code pg 40 table 20.

$\tau_c < \tau_e < T_{u, \max}$ , shear reinforcement is needed providing 25mm $\phi$  r bars on all sides and 2mm $\phi$  2-legged vertical stirrups

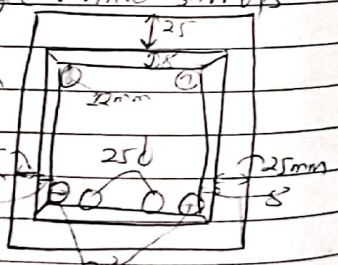
$$A_{sv} = 2 \times \frac{\pi}{4} \times 2^2 = 6.28 \text{ mm}^2$$

$$b_l = 350 - 25 - 25 - 8 - 8 - 28 - 28 = 256 \text{ mm}$$

$$d_l = 210 - 25 - 8 - 12 = 145 \text{ mm}$$

$$x_l = 350 - 25 - 25 - \frac{8}{2} - \frac{8}{2} = 292 \text{ mm}$$

$$y_l = 250 - 25 - 25 - \frac{8}{2} - \frac{8}{2} = 192 \text{ mm}$$



$$S_v = \frac{0.87 f_y A_{sv} d_l}{\frac{T_u}{b_l} + \frac{V_u}{25}} = \frac{0.87 \times 415 \times 6.28 \times 145}{\frac{140 \times 10^6}{256} + \frac{110 \times 10^3}{25}} = 42.41 \text{ mm}$$

NOT OK

providing 12mm $\phi$  2-legged vertical stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 12^2 = 226.194 \text{ mm}^2$$

$$b_l = 248 \text{ mm}, d_l = 167 \text{ mm}, x_l = 288 \text{ mm}, y_l = 188 \text{ mm}$$

$$b_l = 350 - 25 - 25 - 12 - 12 - \frac{28}{2} - \frac{28}{2} = 248 \text{ mm}$$

$$d_l = 210 - 25 - 12 - \frac{12}{2} = 167 \text{ mm}$$

$$x_l = 350 - 25 - 25 - \frac{12}{2} - \frac{12}{2} = 288 \text{ mm}$$

$$y_l = \frac{250 - 25 - 25 - \frac{12}{2} - \frac{12}{2}}{2} = 188 \text{ mm}$$

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Page \_\_\_\_\_

$$S_v = \frac{0.87 \times 415 \times 226.194 \times 167}{\frac{140 \times 10^6}{248} + \frac{110 \times 10^3}{25}} = 89.516 \text{ mm}$$

$$- 89.516 \approx 88 \text{ mm}$$

$$S_v = 88 \text{ mm} < x_l = 288 \text{ mm}$$

$$< \frac{x_l + y_l}{4} = \frac{288 + 188}{4} = 244 \text{ mm}$$

$$< 300 \text{ mm}$$

minimum area of shear reinforcement

$$A_{sv} = (\tau_c - \tau_u) b S_v = (3.018 - 0.608) \times 350 \times 88$$

$$0.87 f_y \quad 0.87 \times 415$$

$$= 205.584 \text{ mm}^2 < 226.194 \text{ mm}^2$$

So provide 12mm $\phi$  2-legged closed vertical stirrups @ 88mm c/c

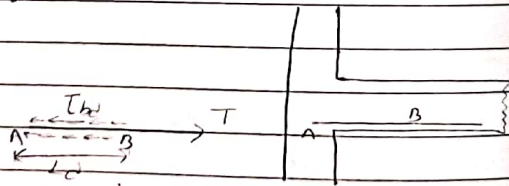
Development length = Bond stress is defined as shear force per unit nominal area of reinforcing bar in the direction parallel to reinforcing bar at interface between reinforcing bar and surrounding concrete.  
Basic assumption is that steel and concrete act together so that there is no slip of reinforcing bar with respect to surrounding concrete.

Two types of bond stress

1) **Anchorage bond** - It is a bond which arises over the length of anchorage provided at ends, near supports and cut off points. This bond prevents steel pulling out of concrete if it is in tension or pushing in of concrete if it is in compression.

2) **Flexural bond** - Flexural bond arises on the account of variation of shear force and bending moment which in turn causes variation in axial tension throughout the length of bar. Flexural bond is critical at a point where shear force  $V = \frac{dM}{dx}$  i.e. variation in bending moment is significant.

**Anchorage bond.**



$L_d$  = development length

$\tau_{bd}$  = bond stress (CL 26.2.1)

For deformed bar, value of  $\tau_{bd}$  given in table is increased by 60% at equilibrium.

Tension in steel = shear force in surrounding steel.

$$\text{or } 0.87 f_y A_{st} = \tau_{bd} \cdot \pi \phi L_d$$

$$\text{or } 0.87 f_y \phi^2 = \tau_{bd} \cdot \pi \phi L_d$$

$$\text{or } L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} \quad \text{[CL 26.2.1 pg 21]}$$

**Flexural bond**

B.M at A,  $M_A = T_A \cdot z$   $z$  = lever arm

B.M at B,  $M_B = T_B \cdot z$

at equilibrium

$$T_B - T_A = \tau_{bd} \cdot \pi \phi dx$$

$$\text{or } \frac{M_B - M_A}{z} = \tau_{bd} \cdot \pi \phi dx$$

$$\text{or } \frac{dM}{dx} = \tau_{bd} \cdot \pi \phi \cdot z \quad \text{where } dM = M_B - M_A$$

$$\text{or } V = \tau_{bd} \cdot \pi \phi \cdot z \quad \text{where shear force } V = \frac{dM}{dx}$$

given.

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} \Rightarrow \tau_{bd} = \frac{0.87 f_y \phi}{4 L_d}$$

$$\text{or } V = \frac{0.87 f_y \phi}{4 L_d} \cdot \pi \phi z$$

$$\text{or } L_d = \frac{0.87 f_y \pi \phi^2 \cdot z}{4 V}$$

$$\text{or } L_d = \frac{0.87 f_y A_{st} \cdot z}{4 V} \quad A_{st} = \pi \phi^2$$

$$L_d = \frac{M}{V} \quad \text{where } M = M_A \text{ or } M_B \text{ in term of steel} = 0.87 f_y A_{st} \cdot z$$

$V$  = factored shear force

$$L_d \leq \frac{M}{V} + L_0$$

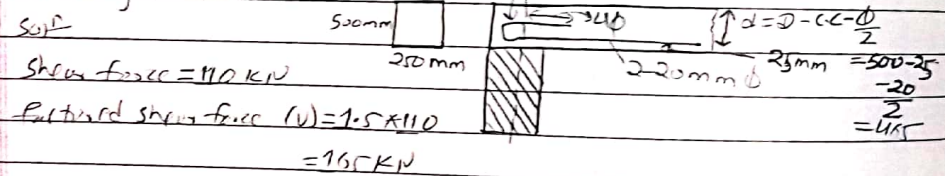
where  $L_0$  = sum of anchorage length beyond the centre of support  
=  $2\phi$  or effective depth 'd' which ever is greater.

max value of  $\mu$  is increased by 30% if reinforcing bar is confined by compressive reaction.

i.e  $L_d \leq 1.3M + L_0$

Bar as mechanical hook	anchorage length ( $L_0$ )
45°	→ 4 $\phi$
90°	→ 8 $\phi$
135°	→ 12 $\phi$
180° U-bend	→ 16 $\phi$

15) Determine the anchorage length for simply supported beams shown. shear force at mid of support is 110 kN. Use M20 mixed grade steel.



Assuming clear cover = 25 mm, effective depth ( $d$ )  $500 - 25 - 20 = 445$  mm

Area of steel =  $2 \times \frac{\pi}{4} \times 20^2 = 628.319$  mm<sup>2</sup>

$X_{u,max} = 0.48d$  for Fe415  
 $= 0.48 \times 445 = 213.6$  mm

$X_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 628.319}{0.36 \times 20 \times 250} = 126.03$  mm < 213.6 mm

Under reinforced section

MOR (M) =  $0.87 f_y A_{st} (d - 0.42 X_u) = 0.87 \times 415 \times 628.319 \times (445 - 0.42 \times 126.03) \times 10^{-6}$   
 $= 93.479$  kNm

$\Gamma_{bd} = 1.2$  provided by IS

$L_d = \frac{0.87 f_y \phi}{4 \Gamma_{bd}} = \frac{0.87 \times 415 \phi}{4 \times 1.2 \times 1.6} = 47.012 \phi = 47 \phi$

i.e  $\Gamma_{bd} = 1.2 \times (1 + 0.6) = 1.2 \times 1.6$

$\Gamma_{bd} = 1.2$  for M20 mix grade steel table 26.2.1

NO. 2,  
 $L_d \leq 1.3M + L_0$  (no hook assumed) best.  
 $\leq 1.3 \times 93.479 = 0.7365 \times 10^3$   
 $165$

or,  $47 \phi \leq 0.7365 \times 10^3$   
 $\phi \leq 15.67$  mm

Since actual dia used is 20 mm, it is not safe in development length providing 90° bend

$L_0 = 8 \phi$   
 $L_d \leq 1.3M + L_0$

or,  $47 \phi \leq 1.3 \times 93.479 \times 10^3 + 8 \phi$   
 $165$

$\phi \leq 18.88$  mm (not ok)

providing U-bend  $L_0 = 16 \phi$   
 Now,  $L_d \leq 1.3M + L_0$

or,  $47 \phi \leq 1.3 \times 93.479 \times 10^3 + 16 \phi$   
 $165$

or,  $\phi \leq \frac{1.3 \times 93.479 \times 10^3}{165 \times (47 - 16)} \leq 23.78$  mm

Since actual dia. provided is 20 mm it is safe in development length arrangement is shown in figure.



23) A simply supported beam  $300\text{cm} \times 500\text{cm}$  is provided 5-20mm  $\phi$  steel bar at bottom at mid span and 316mm  $\phi$  steel bar at top. out of 5-20mm  $\phi$  steel bar at bottom, 2-20mm  $\phi$  bar are bent up at distance 1.5m from face of support. The beam is provided 2-legged 8mm  $\phi$  vertical stirrups at the rate 2-20mm c/c at mid span and 1-20mm  $\phi$  centre to centre near support. Calculate the shear capacity of beam section at support and moment of resistance of beam. Use M20 mix and Fe415 grade steel.

Sol<sup>n</sup>

Assuming 40mm effective cover, effective depth ( $d$ ) =  $500 - 40 = 460\text{mm}$

$$x_{u, \max} = 0.48d = 0.48 \times 460 = 220.8\text{mm}$$

assuming  $x_u < x_{u, \max}$  and  $f_{sc} = 0.87f_y$ ,

$$x_u = \frac{0.87f_y A_{st} - (f_{sc} - 0.446 f_{ck}) A_{sc}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 1570.796 - (353 - 0.446 \times 20) \times 603.185}{0.36 \times 20 \times 300}$$

$$= 164.229\text{mm} < x_{u, \max} = 220.8\text{mm}$$

Under reinforced section.

$$\frac{d'}{d} = \frac{40}{460} = 0.086$$

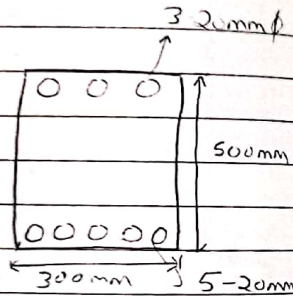
$$\frac{d'}{d} \quad f_{sc}$$

$$f_{sc} = 355 + (353 - 355) \times \frac{0.05}{(0.1 - 0.05)} \rightarrow 355$$

$$0.10 \rightarrow 353$$

$$(0.086 - 0.05) \quad 0.086 \rightarrow 353.56$$

$$= 353.56\text{MPa}$$



$$MOR = 0.36 f_{ck} b x_u (d - 0.42 x_u) + (f_{sc} - 0.446 f_{ck}) A_{sc} (d - d')$$

$$= [0.36 \times 20 \times 300 \times 164.229 \times (460 - 0.42 \times 164.229) + (353.56 - 0.446 \times 20) \times 603.185 \times (460 - 40)] \times 10^{-6}$$

$$= 226.019\text{ kNm}$$

(in 90° inclination theory part)

shear capacity of bent up bar ( $V_{us}$ ) =  $0.87 f_y A_{sv} \sin \alpha$

$$= 0.87 \times 415 \times 628.319 \times \sin 45^\circ$$

$$= 160.410\text{ kN}$$

assuming  $45^\circ$  inclination  $\alpha = 45^\circ$ ,  $A_{sv} = \frac{2 \times \pi \times 20^2}{4} = 628.319\text{mm}^2$

(2 bar bent up at  $45^\circ$ ,  $s = 2 = 3 \times 21$ )

$$\% \text{ of steel} = \frac{A_{st} \times 100}{b \cdot d} = \frac{3 \times \pi \times 20^2 \times 100}{4 \times 2160 \times 300} = 0.683\%$$

design shear stress of concrete ( $\tau_c$ ) =  $0.48 + (0.56 - 0.48) \times \frac{(0.75 - 0.5)}{0.5} = 0.538\text{mm}^2$

code page 40

for M20

$$\frac{0.5}{0.75} \quad 0.48$$

$$0.75 \quad 0.56$$

$$0.683\% = 0.538$$

so shear capacity of concrete ( $V_c$ ) =  $\tau_c b d = 0.538 \times 300 \times 460 \times 10^{-3} = 74.244\text{ kN}$

shear capacity of 2-legged 8mm  $\phi$  vertical stirrups @ 120mm c/c,

$$V_{us} = 0.87 f_y A_{sv} d, \quad A_{sv} = \frac{2 \times \pi \times 8^2}{4} = 100.53\text{mm}^2$$

$$= 0.87 \times 415 \times 100.53 \times 460 = 139.136\text{ kN}$$

120

So, shear capacity of beam =  $V_{us} + V_{uc} + V_{us}$   
 $= 76041 + 74.224 + 1399.36$   
 $= 373.79 \text{ kN}$

18) A R.C.C. beam has an effective depth of 450mm and a breadth of 300mm it contains 5-20mm  $\phi$  TOR steel bar out of which 2-20mm  $\phi$  bar are bent up  $30^\circ$  near support calculate shear capacity of bent-up bar. Use M20 concrete & Fe415 steel grade. What additional vertical stirrups are needed if shear force near support is 125kN as service load.

Area of bent-up bar ( $A_{sv}$ ) =  $2 \times \frac{\pi}{4} \times 20^2$   
 $= 628.319 \text{ mm}^2$

Shear capacity of bent up bar:

$(V_{us}') = 0.87 f_y A_{sv} \sin \alpha$   
 $= 0.87 \times 415 \times 628.319 \times \sin 30^\circ \times 10^{-3}$   
 $= 113423 \text{ kN}$

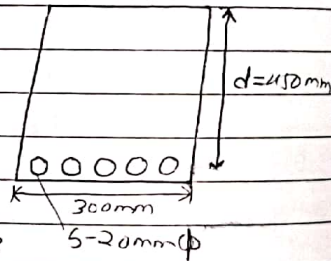
Factored shear force ( $V_u$ ) =  $1.5 \times 125000$   
 $= 187.5 \text{ kN}$

% of steel =  $\frac{A_{st}}{bd} \times 100$

$= \frac{3 \times \pi \times 20^2 \times 100}{4}$

$\frac{450 \times 300}{}$

$= 0.698\%$



design shear stress of concrete ( $\tau_c$ ) =  $0.48 + (0.56 - 0.48) \left( \frac{0.698 - 0.5}{0.75 - 0.5} \right)$

$= 0.543 \text{ N/mm}^2$

shear force to be resisted by shear reinforcement ( $V_{uv}$ ) =  $V_u - V_{uc}$

$= V_u - \tau_c b d$

$= (187.5 - 0.543 \times 300 \times 450) \times 10^{-3}$

$= 114.195 \text{ kN}$

50% of total shear force to be resisted by shear reinforcement is resisted by bent-up bar and 50% by vertical stirrups. So, shear force to be resisted by vertical stirrup ( $V_{us}''$ ) = 50% of 114.195

$= 57.098 \text{ kN}$

Providing 8mm  $\phi$  2-legged vertical stirrups,  $A_{sv} = 2 \times \frac{\pi}{4} \times 8^2$

$= 100.53 \text{ mm}^2$

spacing,  $s_v = \frac{0.87 f_y A_{sv} d}{V_{us}''}$

$= \frac{0.87 \times 415 \times 100.53 \times 450}{57.098 \times 10^3}$

$= 286.058 \approx 285$  (reduce spacing if not done)

$< 300 \text{ mm}$

$< 0.75d = 0.75 \times 450$

$= 337.5$

$\geq 100 \text{ mm OK}$

min area of shear reinforcement =  $0.4 b s_v = 0.4 \times 300 \times 285$   
 $0.87 f_y \quad 0.87 \times 415$

$= 94723 \text{ mm}^2$

$< 100.53 \text{ mm}^2$

So provide 8mm  $\phi$  2-legged vertical stirrups @ 285mm c/c  $\text{OK}$

**Slab:** - slab forms floor and roof of structural building. Generally slab is assumed to carry uniformly distributed load. In most cases, slab is designed for flexure only. Usually slab is horizontal except for staircase and ramp for stored car park. Beam and wall support the slab.

Types of slab

- 1) Simply supported slab spanning in one direction.
- 2) simply supported slab spanning in two direction.
- 3) continuous slab.
- 4) cantilever slab
- 5) flat slab - slab constructed directly over wall.

**Thickness of slab:** - depth of slab is determined from deflection criteria rather than flexural consideration. Deflection should be in permissible limit so that neither appearance nor efficiency is affected. Deflection should not exceed span mm / 250

effective depth of slab = effective span length / (20-26) \* modification factor

modification factor depends upon % of steel and grade of steel (page 7)

effective span to depth ratio,  $l_e$  ratio (code page 7)

cantilever  $\rightarrow 7$   
 simply supported  $\rightarrow 20$   
 continuous  $\rightarrow 26$

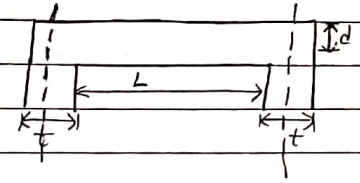
effective span length ( $l_e$ )

$l$  = clear span length.

$t$  = bearing of support.

$d$  = effective depth of slab.

$l_e = L + t$  } - take smaller value.  
 $= l + d$  }



classification of slab

$l_y$  = effective length in longer direction.

$l_x$  = effective length in shorter direction.

1)  $l_y / l_x > 2 \rightarrow$  one way slab.

2)  $l_y / l_x < 2 \rightarrow$  two way slab.

\* Minimum area of steel reinforcement = 0.12% of b D

\* clear cover = 15 mm

\* Spacing  $< 300$  mm (main bar)  
 $< 3d$   
 $> 75$  mm

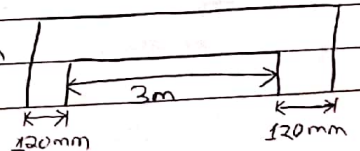
spacing =  $\frac{\text{area of one bar} \times 1000}{\text{total area required}}$   
 distribution bar

slab is assumed 1m or 1000mm wide beam. spacing  $< 5d$   
 $> 450$  mm

19) Design a simply supported slab supported on a masonry wall to the following requirement, bearing on each support equals 120mm clear span 3m, live load 4000 N/m<sup>2</sup> use M20 concrete and Fe415 steel.

[masonry slab over masonry wall]  
 in masonry wall

effective length ( $l_e$ ) = 3 + 0.12 = 3.12 m



$0.58 \times 4.5 = 2.61 \text{ Mmm}$

assuming 0.3% of steel, modification factor = 1.5 (code page 7)  
effective depth of slab (d) = effective span length

simply supported case  

$$= \frac{3120}{20 \times 1.5} = 104$$
 20% modification

assuming 8mm  $\phi$  bar at clear cover 15mm,  $\rightarrow d = d + \text{clear cover} + \frac{\phi}{2}$   
 effective cover =  $15 + \frac{8}{2} = 19$  overall depth =  $104 + 15 + \frac{8}{2} = 123 \text{ mm}$   
 provide 130mm overall depth, effective depth (d) =  $130 - 15 - \frac{8}{2} = 111 \text{ mm}$   
 effective span length (L<sub>e</sub>) = L<sub>t</sub> =  $3 + 0.12 = 3.12 \text{ m}$   
 $= 1 + d = 3 + 0.111 = 3.11 \text{ m}$   
 - twice smaller value  
 $L_c = 3.11 \text{ m}$

Load calculation.

live load =  $4 \times 1 \text{ kNm} = 4 \text{ kNm}$   
 self wt =  $25 \times 1 \times 0.13 \text{ kNm} = 3.25 \text{ kNm}$   
 floor finish (50mm) =  $\frac{24 \times 1 \times 50}{1000} = 1.2 \text{ kNm}$

Total load =  $8.45 \text{ kNm}$

factored load (W<sub>u</sub>) =  $1.5 \times 8.45 = 12.675 \text{ kNm}$

max<sup>m</sup> bending moment (M<sub>u</sub>) =  $\frac{W_u \cdot L_c^2}{8} = \frac{12.675 \times 3.11^2}{8} = 15.334 \text{ kNm}$

check for depth (Design balanced section) (singly reinforce)  
 $M_{max} = 0.36 f_{ck} b x_{u,max} (d - 0.42 x_{u,max})$  (सिंघल रीनफोर्स डिजाइन ऑफ)

or,  $15.334 = 0.36 \times 20 \times 1000 \times 0.48 d (d - 0.42 \times 0.48 d)$

$\times 10^6$  on solving we get

$d = 74.547 \text{ mm} < d = 111 \text{ mm OK}$

area of steel in shorter direction.

$M_{max} = 0.87 f_y A_{st} (d - \frac{f_y A_{st}}{f_{ck} \cdot b})$



or,  $15.334 \times 10^6 = 0.87 \times 415 A_{st} (111 - \frac{415 A_{st}}{20 \times 1000})$

on solving we get

$A_{st} = 414.778 \text{ mm}^2$

provide 8mm  $\phi$  bar at spacing =  $\frac{\pi \times 8^2}{4} \times 1000$   
 $\frac{414.778}{114.778}$

so provide 8mm  $\phi$  bar @ 120mm c/c.

= 121.186 (always reduce)

Actual A<sub>st</sub>, provided = 120 mm spacing (could be 100)  
 $= \frac{\pi \times 8^2}{4} \times 1000$   
 $= 120$

=  $418.879 \text{ mm}^2 > 414.778 \text{ mm}^2 \text{ OK}$

area of distribution bar = 0.12% of b.d

=  $\frac{0.12 \times 1000 \times 130}{100} = 156 \text{ mm}^2$

providing 8mm  $\phi$  bar at spacing =  $\frac{\pi \times 8^2}{4} \times 1000$   
 $\frac{156}{156}$

= 322.214  $\approx 300 \text{ mm}$

provide alternate bar at 1<sup>st</sup> distance from support.

check for shear

shear force (V<sub>u</sub>) =  $\frac{W_u \times \text{clear span}}{2} = \frac{12.675 \times 3}{2} = 19.012$

ultimate shear stress ( $\tau_c$ ) =  $\frac{V_u}{bd} = \frac{19.012 \times 10^3}{1000 \times 104} = 0.182 \text{ N/mm}^2$   
 $= 0.171$

% of steel =  $\frac{A_{st}}{2 \times b \times d} \times 100 = \frac{418.879}{2 \times 1000 \times 104} \times 100$   
 $= 0.2017$  (code page 20)

design shear stress ( $\tau_c$ ) =  $0.76 + (0.36 - 0.28) (0.2017 - 0.15)$   
 $(0.25 - 0.15)$   
 $= 0.79$



$T_c' = k \cdot T_c$  where  $k = 1.3$  for overall depth  $< 150$  mm

$= 1.3 \times 0.39 = 0.51 \text{ N/mm}^2 > 0.182 \text{ N/mm}^2 \text{ OK}$

check for development length.

$MOR (M) = 0.87 f_y A_{st} \left( d - \frac{f_y A_{st}}{2 f_c k b} \right)$

$= 0.87 \times 415 \times 418.879 \left( 111 - \frac{415 \times 418.879}{2 \times 20 \times 1000} \right) \times 10^{-6} \text{ kNm}$

$= 8.064 \text{ kNm}$

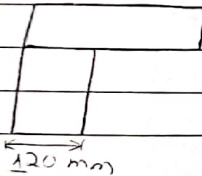
$l_d = \frac{0.87 f_y \phi}{4 T_b d} = \frac{0.87 \times 415 \phi}{4 \times 1.2 \times 1.6} = 47.01 \phi$

$l_{d0} = \text{anchorage length} = \frac{120 - 15}{2} = 45 \text{ mm}$

now,

$l_d \leq 1.3 M + l_{d0}$

or,  $47.01 \phi \leq 1.3 \times 8.064 \times 10^3 + 45$   
 $19.012$



on solving we get

$\phi \leq 12.687 \text{ mm}$

since actual dia of bar used is 8 mm, it is safe in development length.

check for deflection

$\left(\frac{l}{d}\right)_{\text{provided}} = \frac{311}{111} = 2.8027$

$\left(\frac{l}{d}\right)_{\text{permissible}} = K \times \text{basic value}$   
 $\rightarrow 20$

$K = \text{modification factor}$

$f_s = 0.58 f_y$   $\times$  area of steel required  
area of steel provided

$= 0.58 \times 415 \times 418.879$   
 $418.879$

$= 235.343 \text{ Mpa}$

$\% \text{ of steel} = \frac{A_{st} \times 100}{b \cdot d} = \frac{418.879}{1000 \times 111} \times 100 \%$   
 $= 0.377 \%$

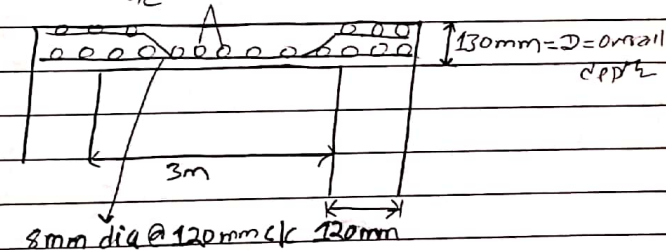
modification factor = 1.4 code page 7 for  $f_c = 238.343 \text{ Mpa}$

$\% = 0.377$

$\left(\frac{l}{d}\right)_{\text{permissible}} = 1.4 \times 20$   
 $= 28 \text{ mm} \geq 28.07 \text{ mm OK}$

$\left(\frac{l}{d}\right)_{\text{provided}}$

8mm  $\phi$  @ 30mm distribution bar



Restrained slab i.e. when corners of slab are prevented from lifting i.e. corners are held down.

for two way slab,  $\frac{l_y}{l_x} \leq 2$

$M_x = \alpha \times w_u \cdot l_x^2$

$M_y = \alpha_y \cdot w_u \cdot l_x^2$

$l_y = \text{Effective length along longer direction}$

$l_x = \text{Effective length along shorter direction}$

$M_y = \text{Max}^m \text{ BM along longer direction}$   $M_x = \text{max}^m \text{ BM along shorter direction per unit width}$

table  
27  
pg 54

$\alpha_x$  = BM coeff along shorter direction.  
 $\alpha_y$  = BM coeff along longer direction. [ref table 26 page 54]

→ longer side ( $L_y$ )

	↓	(1)	(3)	(4)
shorter side ( $L_x$ )	(2)	(1)	(2)	
		(4)	(3)	(4)

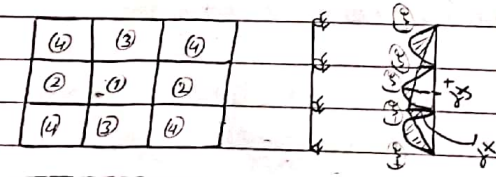
- (1) interior panel (All edge continuous)
- (2) (2) short edge discontinuous
- (3) (2) long edge discontinuous
- (4) (two adjacent edge discontinuous)

→ longer ( $L_y$ )                      → longer ( $L_y$ )

shorter side ( $L_x$ ) ↓	(3)	shorter side ( $L_x$ ) ↓	(5)	(6)	(2)
	(5)				
	(7)				(9)

- (5) Two short edge discontinuous
- (6) Two long edge discontinuous
- (7) Three edge discontinuous (1 edge long continuous)
- (8) Three edge discontinuous (1 short edge continuous)
- (9) Four edge discontinuous      code page 54

table 26



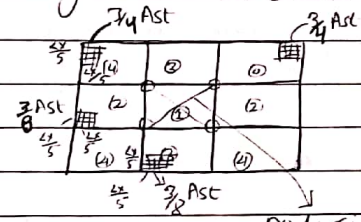
$\alpha_x$  → +ve BM coeff mid span.  
 $\alpha_y$  → -ve BM coeff at continuous support.

provision for torsion.

Torsional reinforcement is provided in both dirn at top and bottom layer with area of steel =  $3 A_{st}$  where  $A_{st}$  = area of steel in mid span for shorter direction.  
with area  $\frac{L_x}{5} \times \frac{L_x}{5}$  where  $L_x$  = effective length in shorter direction.

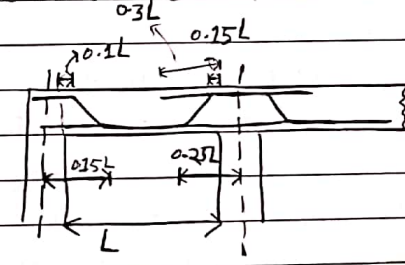
for two edge discontinuous →  $\frac{3}{4} A_{st}$

for one edge discontinuous →  $\frac{3}{8} A_{st}$



for all edge continuous → NO torsional reinforcement.

Detailment rule for reinforcement.



20) Design a interior panel of 4m x 6m continuous slab for a live load of 3 kN/m<sup>2</sup>.  
USE M20 mix and Fe415 steel.

assuming 230mm wide beam, effective length = 4 + 0.23 = 4.23m

effective depth of slab (d) = effective length / 26 × modification factor  
for continuous beam for 415 reinforcement  
20 for simply supported

assuming 0.3% steel, modification factor = 1.5

$$d = 4230 - 26 \times 1.5 = 108.46 \text{ mm}$$

providing 8mm bar at clear cover 15mm, overall depth =  $108.46 + 15 + \frac{8}{2} = 127.46 \text{ mm}$

So provide 130mm overall depth

$$\text{Effective depth (dx)} = 130 - 15 - \frac{8}{2} = 111 \text{ mm}$$

$$\text{effective depth along longer dir}^n (dy) = 111 - 8 = 103 \text{ mm}$$

$$\text{effective span length (Lx)} = 4.0 \text{ m} = 4.0 \text{ m}$$

$$\text{longer dir}^n \quad Ly = 6 + 0.103 = 6.103 \text{ m}$$

Load calculation

$$\text{Live load} = 3 \times 1 = 3 \text{ kN/m}$$

$$\text{Self wt} = 25 \times 1 \times 0.13 = 3.25 \text{ kN/m}$$

$$\text{Floor finish (50mm)} = 24 \times 1 \times 0.05 = 1.2 \text{ kN/m}$$

$$\text{Total load} = 7.45 \text{ kN/m}$$

$$\text{factored load (wu)} = 1.5 \times 7.45 = 11.175 \text{ kN/m}$$

$$\frac{Ly}{Lx} = \frac{6.103}{4.0} = 1.526 < 2 \text{ two way slab}$$

$$\alpha_x^+ = \quad \alpha_y^+ =$$

$$\alpha_x^- = \quad \alpha_y^- =$$

$$\alpha_x^+ = 0.024$$

$$\alpha_y^+ = 0.032$$

$$\alpha_x^- = 0.039 + (0.011 - 0.039) \left( \frac{1.526 - 1.4}{1.5 - 1.4} \right) = 0.0406$$

$$\alpha_x^+ = 1.4 \rightarrow 0.039$$

$$1.5 \rightarrow 0.041$$

$$1.484 \rightarrow 0.0406$$

$$\alpha_x^- = \begin{cases} 1.4 \rightarrow 0.051 \\ 1.5 \rightarrow 0.053 \end{cases}$$

$$1.484 \rightarrow 0.0527$$

Calculation of bending moment

$$M_x^+ = \alpha_x^+ \cdot w_u L_x^2 = 0.0406 \times 11.175 \times 4.0^2 = 7.667 \text{ kNm}$$

$$M_x^- = \alpha_x^- \cdot w_u L_x^2 = 0.0527 \times 11.175 \times 4.0^2 = 9.953 \text{ kNm}$$

$$M_y^+ = \alpha_y^+ \cdot w_u L_y^2 = 0.024 \times 11.175 \times 6.103^2 = 4.533 \text{ kNm}$$

$$M_y^- = \alpha_y^- \cdot w_u L_y^2 = 0.039 \times 11.175 \times 6.103^2 = 6.043 \text{ kNm}$$

check for depth

$$M_{max} = 0.36 f_c k \cdot b \cdot x_{u,max} (d - 0.42 x_{u,max})$$

at max value of  $x_u$

$$\text{or, } 9.953 \times 10^6 = 0.36 \times 170 \times 1000 \times 0.48 d (d - 0.42 \times 0.48 d)$$

On solving we get

$$d = 111 \text{ mm (at } x_{u,max} \text{ compare } d) \\ d = 60.06 \text{ mm} < 111 \text{ mm OK}$$

Calculation of area of steel

$$M = 0.87 f_y A_{st} (d - \frac{f_y A_{st}}{f_c k b})$$

$$\text{For } M_x^+ = 7.667$$

$$\text{for } M_x^- = 9.953$$

$$\text{for } M_y^+ = 4.533 \quad \text{for } M_y^- = 6.043$$

$$A_{stx}^+ = 148.688 \text{ mm}^2$$

$$A_{stx}^- = 261.093 \text{ mm}^2$$

$$A_{sty}^+ = 125.044 \text{ mm}^2 \quad A_{sty}^- = 168.197 \text{ mm}^2$$

eg

$$7.667 \times 10^6 = 0.87 \times 415 \times A_{st} (111 - \frac{415 \times A_{st}}{20 \times 1000})$$

$$4.533 \times 10^6 = 0.87 \times 415 \times A_{st}$$

On solving

effective depth

$$\times (103 - \frac{415 \times A_{st}}{20 \times 1000})$$

$$A_{stx}^+ = 148.688 \text{ mm}^2$$

in x-dir

On solving

effective depth in y-dir

$$A_{stx}^- =$$

$$9.953 \times 10^6 = 0.87 \times 415 \times A_{st} (111 - \frac{415 \times A_{st}}{20 \times 1000})$$

$$A_{stx}^- = 261.093 \text{ mm}^2$$

Minm area of steel = 0.12% of  $b \times D$   

$$= \frac{0.12 \times 1000 \times 130}{100}$$

$$= 156 \text{ mm}^2$$

provide 8mm  $\phi$  bar at spacing =  $\frac{\pi \times 8^2}{4} \times 1000$   
 $A_{st}^+ = 198.688$   
 $= 252.987 \text{ mm} \approx 240 \text{ mm}$  (reduce spacing)

provide 8mm  $\phi$  bar at spacing =  $\frac{\pi \times 8^2}{4} \times 1000$   
 $A_{st}^+ = 261.093$   
 $= 192.99 \text{ mm} \approx 190 \text{ mm}$

provide 8mm  $\phi$  bar at spacing =  $\frac{\pi \times 8^2}{4} \times 1000$   
 $A_{st}^+ = 156$   
 $= 322.215 \text{ mm} \approx 330 \text{ mm}$   
 must not exceed minm value of 156mm  
 No. P125.044 < 156.50, 156mm<sup>2</sup> is used

provide 8mm  $\phi$  bar at spacing =  $\frac{\pi \times 8^2}{4} \times 1000$   
 $A_{st}^+ = 168.197$   
 $= 298.849 \text{ mm} \approx 290 \text{ mm}$

Actual  $A_{st}$  provided =  $\frac{\pi \times 8^2}{4} \times 1000 \text{ mm}^2$   
 $= 209.439 \text{ mm}^2$   
 $> 198.688 \text{ mm}^2$   
 Always use  $A_{st}^+$  while checking (shorter side) पर छोटा जाँच  
 [shorter side mid span] पर छोटा जाँच  
 [shorter side at critical] पर छोटा जाँच  
 OK  $A_{st}^+ = 198.688 \text{ mm}^2$  पर 240 लेंगे ही। (shorter span  $\rightarrow$  critical)



check for shear  
 shear force ( $V_u$ ) =  $w_u \times \text{clear span} = \frac{11.175 \times 4}{2} = 22.35 \text{ kN}$

Ultimate shear stress ( $T_u$ ) =  $\frac{V_u}{bd} = \frac{22.35 \times 10^3}{1000 \times 111} = 0.201 \text{ N/mm}^2$

% of steel =  $\frac{A_{st}^+}{bd} \times 100\%$   
 $= \frac{209.439 \times 100}{1000 \times 111} = 0.188\%$   
 [if spacing 3mm and given so divided by 2 जाँचें]

Design shear stress ( $T_c$ ) =  $0.28 + 0.36 - 0.28 \times (0.188 - 0.15)$   
 $= 0.31$  code pg 40 table 19 for

$T_c' = k \cdot T_c$  where  $k = 1.3$  for  $D = 150 \text{ mm}$  M20 & % of steel = 0.188%  
 $= 1.3 \times 0.31 = 0.403 \text{ N/mm}^2 > 0.201 \text{ N/mm}^2$  OK  
 (safe no need to add stirrups)

check for development length,  $MOR = (M) = 0.87 f_y A_{st}^+ (d - f_y A_{st}^+)$   
 for b

$= 0.87 \times 415 \times 209.439 \times (111 - 415 \times 209.439)$   
 $= 209.439 \times 20 \times 1000$   
 $\times 10^{-6}$

$l_d = \frac{0.87 f_y \phi}{4 T_c} = \frac{0.87 \times 415 \phi}{4 \times 0.31} = 47.01 \phi \approx 47 \phi$   
 add extra 0.01. code pg 12

$l_d \leq 1.3 M + 2 \phi$   
 first check by making  $l_d = 0$  if it is unsafe then use  $l_d$  value.

$\alpha_s 47 \phi \leq \frac{1.3 \times 8.064 \times 10^3}{22.35}$





Since actual dia of bar provided is 8mm, it is safe in development length  
 $< 9.97 \text{ mm}$   
OK

Check for deflection

$(\frac{d}{l})$  provided =  $\frac{4111}{32036} = 0.128$

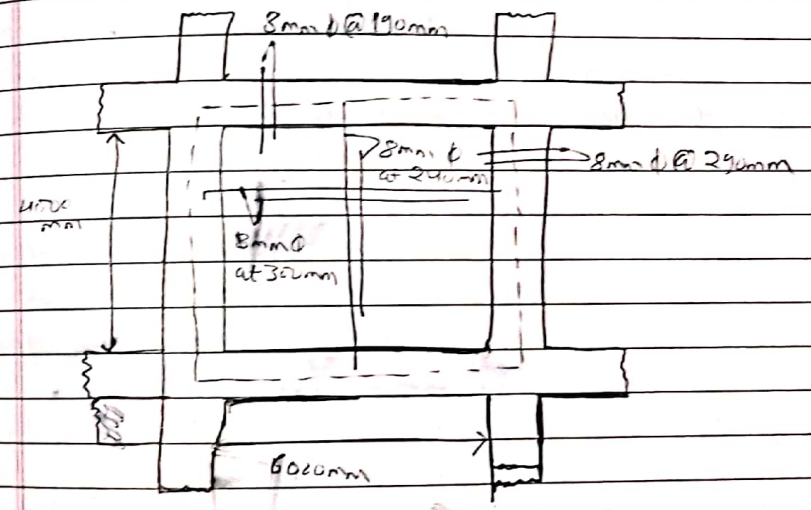
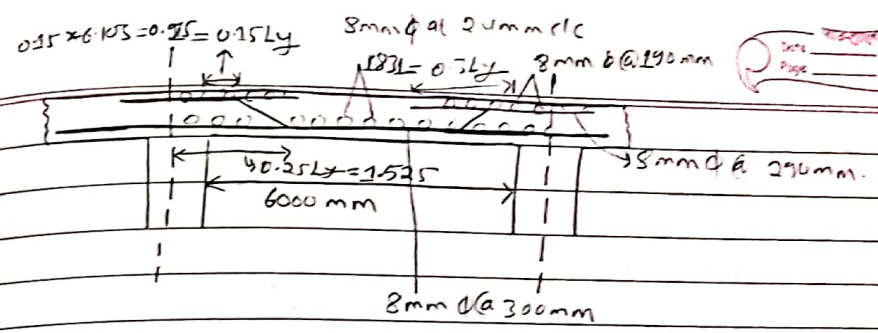
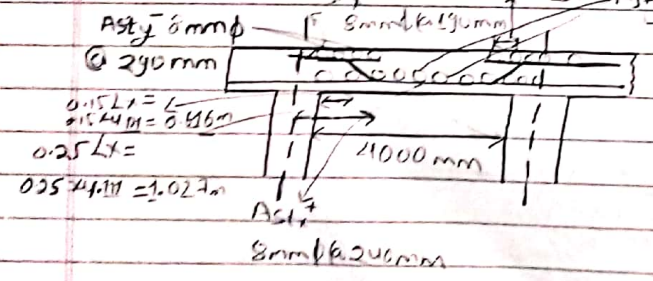
$(\frac{d}{l})$  permissible =  $k \times \text{base value}$   
 $k = \text{modification factor}$

$f_s = 0.58 f_y \times \frac{\text{area of steel required}}{\text{area of steel provided}}$   
 $= 0.58 \times 415 \times \frac{148.688}{209.439}$   
 $= 228.344 \text{ Mpa}$

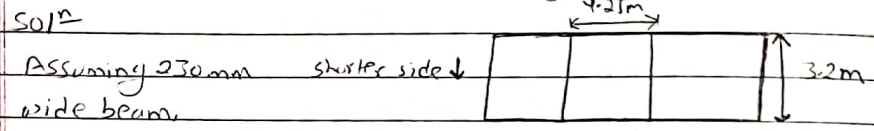
$\therefore$  % age of steel = 0.128%

Now, from code page 7 draft  
 $k = 1.75$

$(\frac{d}{l})$  permissible =  $1.75 \times 0.26 = 45.5 > 37.036$  OK  
 $A_{st} = 0.32 \times \frac{0.23 \times 4111}{230} = 230 \text{ mm}^2$   
 $A_{st} = 8 \text{ mm } \phi$  5mm @ 190mm + 8mm @ 300mm



21. Design a two long edge discontinuous slab with (clear) 3.2m side 3.2m x 4.25m to support live load of 2 kN/m<sup>2</sup>, 40mm RCC floor mix with finishing and 1 kN/m<sup>2</sup> partition wall load. Use M20 and FE415 steel.



Assuming 230mm starter side wide beam.

Effective length = 3.2 + 0.23 = 3.43m

effective depth of slab (d) = effective length / 23 \* modification factor.

$$= \frac{3430}{23 \times 1.5} = \frac{20 + 26}{2} = 23$$

longer side is not continuous

shorter side is not simply supported so average value taken

d = 99.42mm

providing 8mm dia bar of 15mm clear cover, overall depth = 99.42 + 15 + 8 = 122.42mm

provide 125mm overall depth

effective depth along shorter span (dx) = 125 - 15 - 8 = 102mm

effective depth along longer span (dy) = 102 - 8 = 94mm

effective length along shorter span (Lx) = 3.2 + 0.106 = 3.306m

effective length along longer span (Ly) = 4.25 + 0.098 = 4.348m

load calculation.

live load = 2 x 3 = 2 kN/m

self wt = 25 x 1 x 0.125 = 3.125 kN/m

floor finishing (40mm) = 24 x 1 x 0.04 = 0.96 kN/m

partition wall load = 1 x 1 = 1 kN/m

Total load = 7.085 kN/m



factored load (wu) = 1.5 x 7.085 = 10.627 kN/m

$\frac{L_y}{L_x} = \frac{4.348}{3.306} = 1.315 < 2$ , two way slab

$\alpha_x = 0.0579 = 0.058$

$\alpha_y = 0.035$

1.3 → 0.057

$\alpha_y = 0.045$

1.4 → 0.063

1.315 → 0.0599

code pg 54 table 26

calculation of bending moment.

$M_x^+ = \alpha_x \times w_u \times L_x^2 = 0.058 \times 10.627 \times 3.306^2 = 6.736 \text{ kNm}$

$M_y^+ = \alpha_y \times w_u \times L_x^2 = 0.035 \times 10.627 \times 3.306^2 = 4.065 \text{ kNm}$

$M_y^- = \alpha_y \times w_u \times L_x^2 = 0.045 \times 10.627 \times 3.306^2 = 5.226 \text{ kNm}$

check for depth.

$M_{max} = 0.36 f_{ck} \cdot b \cdot x_{u,max} (d - 0.42 x_{u,max})$

or, 6.736 = 0.36 x 20 x 1000 x 0.48d (d - 0.42 x 0.48d)

On solving we get d = 49.1108mm < dx = 106mm OK

calculation of area of reinforcement.

minimum Ast to be provided Ast,min = 0.12% of b.D

= 0.12 x 1000 x 125 = 150 mm<sup>2</sup>

calculation of area of steel.

$A_s = 0.87 f_y A_{st} \left( \frac{d - f_y A_{st}}{f_{ck} \cdot b} \right)$



d=106mm

d=98mm

d=98mm

$M_x^+ = 6.736 \text{ kNm}$

$M_y^+ = 11.065 \text{ kNm}$

$M_j = 5.226 \text{ kNm}$

$A_{stx}^+ = 182.528 \text{ mm}^2$

$A_{sty}^+ = 117.825 \text{ mm}^2$

$A_{sj} = 152.631 \text{ mm}^2$

spacing =  $\frac{\pi \times R^2 \times 1000}{A_{stx}^+}$   
 $\frac{182.528}{182.528}$

spacing =  $\frac{\pi \times R^2 \times 1000}{A_{sty}^+}$   
but =  $\frac{\pi \times 22^2 \times 1000}{117.825}$

spacing =  $\frac{\pi \times R^2 \times 1000}{A_{sj}}$   
 $\frac{152.631}{152.631}$

$\approx 275.385 \text{ mm}$   
 $\approx 175 \text{ mm}$

$\approx 335.103 \text{ mm}$   
 $\approx 270 \text{ mm}$

$\approx 329.326 \text{ mm}$   
 $\approx 230 \text{ mm}$

along shorter dir<sup>n</sup> at mid span provide 8mm  $\phi$  bar at 175mm/c/c  
along longer dir<sup>n</sup> at mid span provide 8mm  $\phi$  bar at 270mm/c/c  
along longer dir<sup>n</sup> at continuous support provide 8mm  $\phi$  bar at 230mm/c/c

Actual  $A_{st}$  provided =  $\frac{\pi \times 8^2}{4} \times 1000$

$A_{stx}$  at mid span is 175 use 7mm =  $287.231 \text{ mm}^2 > 182.528 \text{ mm}^2 \text{ OK}$

check 50% of bar provided at mid span ( $A_{stx}^+$ )

check for shear. to the support.

shear force ( $V_u$ ) =  $\frac{\text{clear span} \times w_u}{2} = \frac{10.627 \times 3.2}{2}$

$= 17.003 \text{ kN}$

ultimate shear stress ( $T_u$ ) =  $\frac{V_u}{bd} = \frac{17.003 \times 10^3}{1000 \times 106} = 0.16 \text{ N/mm}^2$

% of steel =  $\frac{A_{stx}^+}{2bd} \times 100 = \frac{287.231 \times 100}{2 \times 1000 \times 106}$  (2mm spacing wire divided by 2 mm dia)  
 $= 0.135\%$

$T_c = 0.28 \text{ N/mm}^2$

for solid slabs  $T_c' = k T_c$  where  $k = 1.3$  for  $D < 150 \text{ mm}$

$= 1.3 \times 0.28$

$= 0.364 \text{ N/mm}^2 > 0.16 \text{ N/mm}^2 \text{ OK}$

check for development length.

$l_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 415 \phi}{4 \times 1.2 \times 1.6}$   
 $\approx 100 \text{ mm}$

$MOR (M) = 0.87 f_y \frac{A_{stx}^+}{2} \left( \frac{d - f_y A_{stx}^+}{2 f_k b} \right)$

or  $M = \frac{0.87 \times 415 \times 287.231}{2} \left( \frac{106 - 415 \times 287.231}{2 \times 20 \times 1000} \right) \times 10^{-6} \text{ kNm}$   
 $= 5.341 \text{ kNm}$

Now  $l_d \leq 1.3 M + \frac{10}{V}$

or  $47 \phi < 1.3 \times 5.341 \times 10^3 + 17.003$

or  $\phi < 8.688 \text{ mm}$

Since actual dia of bar provided is 8mm, it is safe in development length.  $< 8.688 \text{ mm}$

check for deflection.

$\left( \frac{l}{d} \right)_{\text{provided}} = \frac{5306}{31188}$

$\left( \frac{l}{d} \right)_{\text{permissible}} = k \times \text{base value}$

$k =$  modification factor.

% of steel =  $\frac{A_{stx}^+}{bd} \times 100 = 0.27\%$

$$f_s = 0.58 f_y \times \frac{\text{area of steel required}}{\text{area of steel provided}}$$

$$= 0.58 \times 415 \times \frac{182.528}{287.231} = 152.958 \text{ MPa}$$

$K=2$  from code page 7.  $\therefore$   $\rho_{st} = 0.27\%$

$$f_s = 152.958$$

( $\frac{L}{d}$ ) provided

(d) permissible  $- 2 \times 23 = 46 > 348.8 \text{ OK}$

Area of steel for torsion =  $\frac{3}{8} A_{st} x^t$

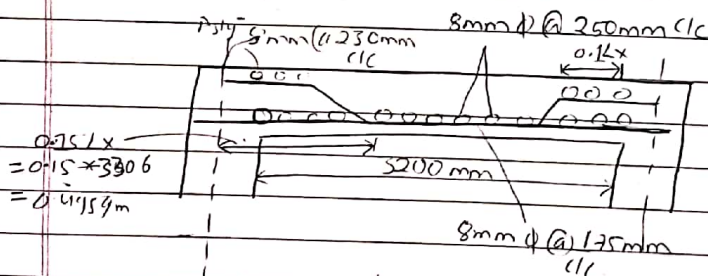
$$= \frac{3}{8} \times 287.231 = 107.712 \text{ mm}^2$$



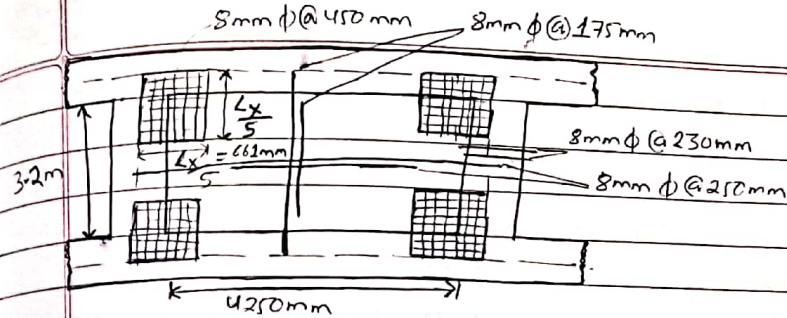
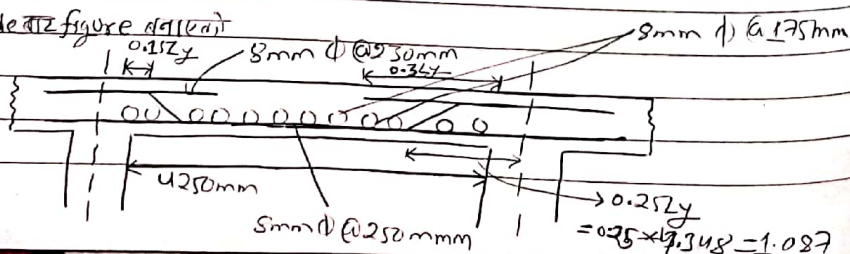
Provide 8mm  $\phi$  bar at spacing  $\frac{x^2}{4} \times \frac{1000}{107.712} = 466.65 \approx 450 \text{ mm}$

in both dir<sup>n</sup> at top and bottom

shorter side figure nikalo



longer side figure nikalo



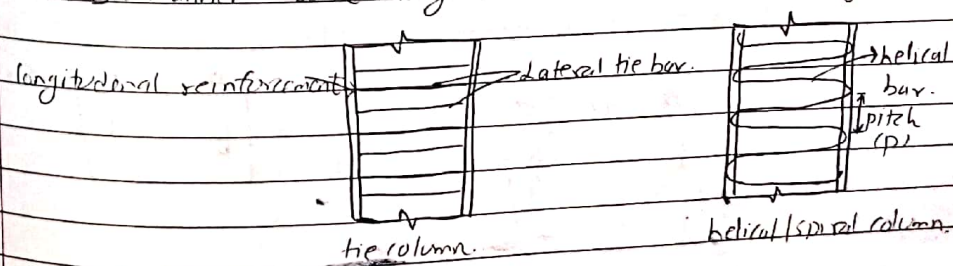
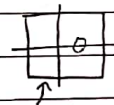
2021 December 22

Chapter - four. Design of column and footing 30marks

Column - Column is an important component of structural element. Column supports beam which in turn supports wall and slab. Column is a compression member whose longitudinal dimension exceeds three times lateral dimension. Compression member whose longitudinal dimension does not exceed three times its lateral dimension is known as pedestal.

Classification of column

- \* Based on types of loading
  - a) axially loaded column.
  - b) column subjected to axial load and uniaxial moment.
  - c) column subjected to axial load and biaxial moment.
- \* Based on manner of which longitudinal reinforcement is laterally supported



\* Based on slenderness ratio

$$\text{Slenderness ratio } (\lambda) = \frac{L_{\text{eff}} \text{ or } L_{\text{eff}}}{d \quad b}$$

$L_{\text{eff}}$  = effective length along major axis

$L_{\text{eff}}$  = effective length along minor axis

$d$  = lateral dimension along major axis (greater lateral dimension)

$b$  = lateral dimension along minor axis (smaller lateral dimension)

i)  $\lambda \leq 12 \rightarrow$  short column

ii)  $\lambda > 12 \rightarrow$  long column

effective length of column (table 28, page 55)

Minimum eccentricity,  $e_{\text{min}}$

$$e_{\text{min}} = \frac{L}{500} + \frac{b}{30}$$

$$= \frac{L}{500} + \frac{d}{30}$$

$$= 20 \text{ mm}$$

} greater value

} should be adopted

(all dimension in mm)

$L$  = unsupported length in mm

$b$  = least lateral dimension in mm

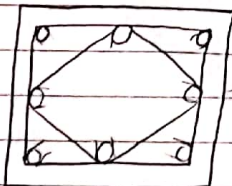
$d$  = greatest lateral dimension in mm

\* clear cover 40 mm

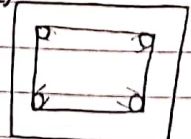
\* % of steel (0.8-6%) upto 4% is preferred

% of steel of gross area.

lateral reinforcement arrangement (page 17, 18, 19)



for 8 bar



for 4 bar

diameter of lateral tie bar

$$\phi > 6 \text{ mm}$$

$$= \frac{1(\phi_L)_{\text{max}}}{4}$$

$(\phi_L)_{\text{max}}$  = greatest diameter of longitudinal reinforcement.

spacing of lateral tie bar  $< 300 \text{ mm}$

$\leq$

$$< 16(\phi_L)_{\text{min}}$$

$$< 48\phi_L$$

$(\phi_L)_{\text{min}}$  = smallest dia. of longitudinal reinforcement.

$\phi_L$  = dia. of lateral tie bar.

Axially loaded short column.

If minimum eccentricity,  $e_{\text{min}} < 0.05b$ ,  $b$  = least lateral dimension

$$P_u = 0.4f_c k A_c + 0.67 f_y A_{sc}$$

where  $A_c$  = area of concrete =  $A_g - A_{sc}$

$A_g$  = gross area =  $bd$

$A_{sc}$  = area of steel.

$P_u$  = factored axial load.

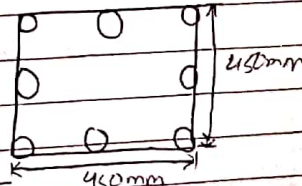
22) A short RCC column  $450 \text{ mm} \times 450 \text{ mm}$  is reinforced with 8-20 mm bar. The unsupported length of column is 2.75 m. Find the ultimate load for column. Use M20 concrete and Fe250 steel.

Sol<sup>n</sup>

unsupported length ( $L$ ) = 2.75 m

$$\text{Area of steel } (A_{sc}) = \frac{8 \times \pi \times 20^2}{4}$$

$$= 2513.274 \text{ mm}^2$$



$$\text{area of concrete } (A_c) = A_g - A_{sc} = 450 \times 450 - 2513.274$$

$$= 199986.725 \text{ mm}^2$$

$$e_{min} = \frac{L}{500} + \frac{b}{30} = \frac{2750}{500} + \frac{450}{30} = 20.5 \text{ mm}$$

} greater value adopted  
 $= 20 \text{ mm}$   $\therefore e_{min} = 20.5 \text{ mm}$   
 $0.05b$

$$= 0.05 \times 450$$

$$= 22.5 \text{ mm} \geq e_{min}$$

So, ultimate load,  $P_u = 0.4 f_c k A_c + 0.67 f_y A_{sc}$

$$= 0.4 \times 20 \times 199986.725 + 0.67 \times 250 \times 2513.274$$

$$\times 10^{-3} \text{ kN}$$

$$= 2020.867 \text{ kN}$$

23) A R.C.C column 450mm x 450mm has to carry factored axial load of 1800 kN. The unsupported length of column is 2m. Find the amount of reinforcement required. Use M20 concrete and Fe25 steel.

Soln

factored axial load ( $P_u$ ) = 1800 kN 400 लेटा कर के देना!

$$e_{min} = \frac{L}{500} + \frac{d}{30} = \frac{2000}{500} + \frac{450}{30} = 19 < 20$$

$$e_{min} = 20 \text{ mm}$$

$$0.05b = 0.05 \times 450 = 22.5 \text{ mm} \geq e_{min}$$

$$\text{area of concrete } (A_c) = A_g - A_{sc} = 450 \times 450 - A_{sc}$$

$$= 180000 - A_{sc}$$

Using relation,

$$P_u = 0.4 f_c k A_c + 0.67 f_y A_{sc}$$

$$1800 \times 10^3 = 0.4 \times 180000 - A_{sc} \times 20 + 0.67 \times 250 \times A_{sc}$$

or solving we get,

$$A_{sc} = 2257.053 \text{ mm}^2$$

provide 8-20mm  $\phi$  longitudinal bar.

$$\text{area of steel provided} = 8 \times 311 = 2488 \text{ mm}^2 > 2257.053 \text{ mm}^2$$

$OK$

Design of lateral reinforcement

dia. of lateral tie bar

$$\phi \geq 6 \text{ mm}$$

$$\geq \frac{(\phi L)_{max}}{4} = \frac{20 \times 2750}{4} = 1375 \text{ mm}$$

provide 6mm  $\phi$  lateral tie bar.

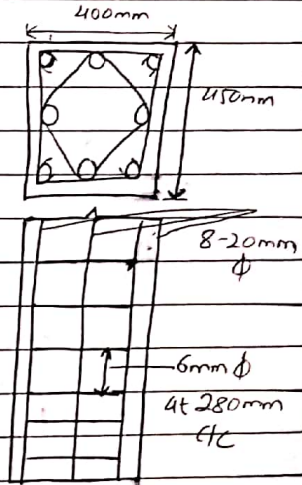
spacing  $< 300 \text{ mm}$ .

$$< b = 400 \text{ mm}$$

$$< 16(\phi L)_{max} = 16 \times 20 = 320 \text{ mm}$$

$$< 4.8 \phi L = 4.8 \times 6 = 288 \text{ mm}$$

So, provide 6mm  $\phi$  lateral tie bar @ 280mm c/c.



24) A reinforced concrete column of 2.35m length carries an axial load of 1600 kN. Design the column using M20 concrete and Fe25 steel.

Soln

factored axial load ( $P_u$ ) =  $1.5 \times 1600 = 2400 \text{ kN}$

assume 2% of steel (0.2-0.4)

$$\text{area of steel } (A_{sc}) = 2\% \text{ of gross area } (A_g) = 0.02 A_g$$

$$\text{area of concrete } (A_c) = A_g - A_{sc} = A_g - 0.02 A_g = 0.98 A_g$$

assuming  $e_{min} \leq 0.05b$

$$P_u = 0.4 f_c k A_c + 0.67 f_y A_{sc}$$

$$2400 \times 10^3 = 0.4 \times 20 \times 0.98 A_g + 0.67 \times 250 \times 0.02 A_g$$

$$\frac{L}{d} < 12$$

$$d > \frac{L}{12} = \frac{2750}{12} = 229.162 \text{ mm}$$

7th and 1st of steel calculation  
5th L 2nd 2nd 1st

$$or, 2400 \times 10^3 = 0.4 \times 20 \times 0.98 A_g + 0.67 \times 415 \times 0.02 A_g$$

on solving we get.

$$A_g = 179091.1126 \text{ mm}^2$$

assuming square column, side =  $b = d = \sqrt{A_g} = \sqrt{179091.1126}$   
 $= 423.192 \text{ mm}$

So, provide 430mm x 430mm column.

area of steel,  $A_{sc} = 0.02 A_g = 0.02 \times 430 \times 430$   
 $= 3698 \text{ mm}^2$

provide 8-25mm  $\phi$  longitudinal bar, ( $A_{sc}$ ) provided =  $8 \times 490$   
 $= 3920 \text{ mm}^2$

design of lateral tie bar.

dia ( $\phi$ ) = 6mm.

$$\geq \frac{(\phi L)_{\max}}{4} = \frac{25}{4} = 6.25 \text{ mm}$$

$$e_{\min} = \frac{L}{500} + \frac{b}{30} = \frac{2750}{500} + \frac{430}{30}$$

$$= 19.83 \text{ mm}$$

provide 8mm  $\phi$  tie bar.

spacing

$$< 300 \text{ mm}$$

$$< b = 430 \text{ mm}$$

$$< 16(\phi L)_{\min} = 16 \times 25 = 400 \text{ mm}$$

$$e_{\min} = 20 \text{ mm}$$

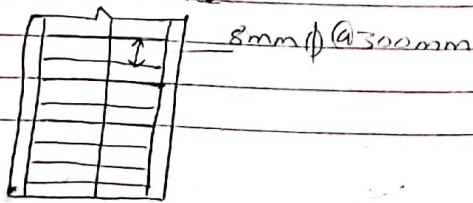
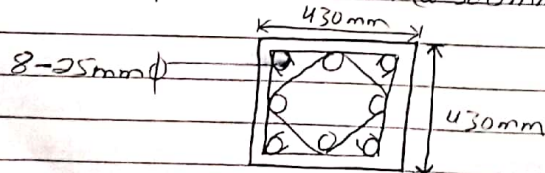
$$0.05b = 0.05 \times 430$$

$$= 21.5 \text{ mm} > e_{\min}$$

OK

$$< 48\phi_L = 48 \times 8 = 384 \text{ mm}$$

So, provide 8mm  $\phi$  lateral tie bar @ 300mm c/c



Spirally reinforced circular column

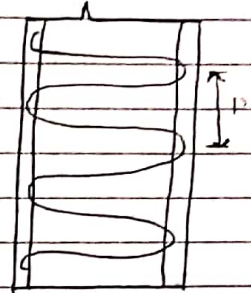
$$P_u = 1.05 (0.4 f_{ck} A_c + 0.67 f_y A_{sc})$$

To satisfy the above relation.

Volume of spiral reinforcement

Volume of core per pitch length.

$$= 0.36 \left( \frac{A_g - 1}{A_c} \right) \frac{f_{ck}}{f_y}$$



$A_g$  - gross Area.

$A_c$  - area of core =  $\frac{\pi}{4} d_c^2$

$d$  - dia. of core

$d_c = d - 2 \times \text{clear cover}$ .

Volume of spiral reinforcement =  $\frac{\pi}{4} \times \phi_{sp}^2 \times p \times \pi d_c$



Volume of core =  $\frac{\pi}{4} d_c^2 \times p$

dia of spiral reinforcement.

$$\phi \geq 6 \text{ mm}$$

$$> \frac{(\phi L)_{\max}}{4}$$

pitch (p)  $> 25 \text{ mm}$

$$< 75 \text{ mm}$$

(code pg 43)

$$< \frac{d_c}{6}$$

$$> 3\phi_{sp}$$

$\phi_{sp}$  = dia. of spiral reinforcement

25) Design a circular column to carry axial load of 1500 kN. Using lateral tie & helical reinforcement. Use M25 and Fe415 steel.

i) factored axial load (Pu) = 1.5 × 1000 = 2250 kN.

Assuming 0.8% of steel (0.8-6%)

$$A_{sc} = 0.8\% \text{ of } A_g = 0.008 A_g$$

$$A_c = A_g - A_{sc} = A_g - 0.008 A_g = 0.992 A_g$$

now,

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$0.2250 \times 10^3 = 0.4 \times 25 \times 0.992 A_g + 0.67 \times 415 \times 0.008 A_g$$

on solving

$$A_g = \frac{185270.577 \text{ mm}^2}{4} = 46317.644 \text{ mm}^2$$

$$\text{dia } D = \sqrt{\frac{4 A_g}{\pi}} = \sqrt{\frac{4 \times 46317.644}{\pi}} = 242.69 \text{ mm}$$

provide 500 mm  $\phi$  circular column.

$$A_{sc} = 0.008 \times \pi \times 500^2 = 1570.796 \text{ mm}^2$$

provide 8-16 mm  $\phi$  bar.

(Circular  $\phi$  at least min)  
8  $\phi$  bar  $\phi$  16 mm

$$A_{sc, \text{ provided}} = 8 \times 201 = 1608 \text{ mm}^2 > 1570.796 \text{ mm}^2$$

8-16 mm  $\phi$

Design of lateral tie bar.

$$\text{dia } \phi \geq 6 \text{ mm}$$

$$\geq \frac{\phi_{\text{max}}}{4} = \frac{16}{4} = 4 \text{ mm}$$

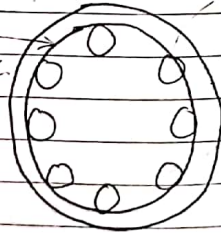
provide 6 mm  $\phi$  lateral tie bar.

$$\text{Spacing} < 500 \text{ mm}$$

$$< D = 500 \text{ mm}$$

$$< 16 \phi_{\text{min}} = 16 \times 16 = 256 \text{ mm} < 250 \text{ mm}$$

$$< 48 \phi = 48 \times 6 = 288 \text{ mm}$$



so provide 6 mm  $\phi$  tie bar @ 250 mm c/c

ii) factored axial load (Pu) = 1.5 × 1500 = 2250 kN.

assuming 0.8% of steel

$$A_{sc} = 0.8\% \text{ of } A_g = 0.008 A_g$$

$$A_c = A_g - A_{sc} = A_g - 0.008 A_g = 0.992 A_g$$

now,

$$P_u = 1.05 (0.4 f_{ck} A_c + 0.67 f_y A_{sc})$$

$$0.2250 \times 10^3 = \frac{1.05}{1.05} (0.4 \times 25 \times 0.992 A_g + 0.67 \times 415 \times 0.008 A_g)$$

on solving,

$$A_g = \frac{176448.168 \text{ mm}^2}{4} = 44112.042 \text{ mm}^2$$

$$\text{dia } D = \sqrt{\frac{4 A_g}{\pi}} = \sqrt{\frac{4 \times 44112.042}{\pi}} = 237.983 \text{ mm}$$

provide 480 mm  $\phi$  circular column

$$A_{sc} = 0.008 \times \pi \times 480^2$$

$$= 1442.645 \text{ mm}^2$$

provide 8-16 mm  $\phi$  bar.

$$A_{sc, \text{ provided}} = 8 \times 201 = 1608 \text{ mm}^2 > 1442.645 \text{ mm}^2$$

Design of helical bar dia  $\phi \geq 6 \text{ mm}$

$$\geq \frac{\phi_{\text{max}}}{4} = \frac{16}{4} = 4 \text{ mm}$$

Satisfy  $\phi$  increase  $\phi$  if max core increase  $\phi$

provide 6 mm  $\phi$  helical bar

To satisfy the above condition

$$\text{Vol}^m \text{ of spiral reinforcement} = 0.36 \left( \frac{A_g - 1}{A_c} \right) f_{ck}$$

$$\text{Vol}^m \text{ of core per pitch length} = \frac{A_g - 1}{A_c} f_y$$



assuming 40mm clear cover

$$\text{dia of core } (d_c) = D - 2 \times \text{clear cover} \\ = 480 - 2 \times 40 = 400 \text{ mm}$$

$$\frac{\pi \phi_s^2 \times n \times d_c}{4 \times d_c^2 \times p} \geq 0.36 \left( \frac{\pi D^2}{4} - 1 \right) \frac{f_k}{f_y}$$

$$\text{or, } \frac{6^2 \times \pi}{400 \times p} \geq 0.36 \left( \frac{480^2}{4} - 1 \right) \frac{25}{415}$$

$$\text{or, } \frac{0.2227}{p} \geq 9.542 \times 10^{-3}$$

$$p \leq 29.62 \text{ mm}$$

from code, pitch  $p \geq 25$

$$\leq 75 \text{ mm}$$

$$< d_c/6 = \frac{400}{6} = 66.667 \text{ mm}$$

$$\geq 3\phi_s = 3 \times 6 = 18 \text{ mm}$$

So provide 6mm  $\phi$  helical bar at pitch distance 25mm c/c.

26) Determine the reinforcement in spiral column 400mm dia subjected to factored load 1500 kN. The column has unsupported length of 3.4m. Use M25 & Fe415 steel.

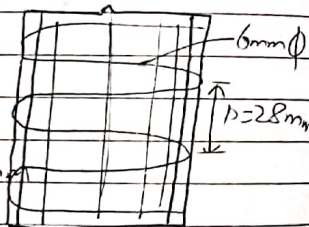
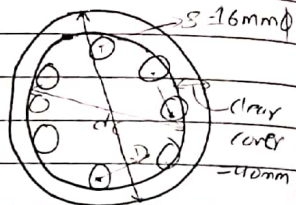
Sol<sup>n</sup>

factored load ( $P_u$ ) = 1500 kN.

$$e_{min} = \frac{L}{50} + \frac{D}{30} = \frac{3400}{50} + \frac{400}{30} = 201.33 > 20 \text{ mm}$$

$$e_{min} = 20.13 \quad (\text{take greater value}) \quad A_c = A_g - A_{sc}$$

$$0.05D = 0.05 \times 400 = 20 \text{ mm} \approx e_{min}$$



Normal

$$P_u = 1.05 (0.4 f_{ck} A_c + 0.67 f_y A_{sc})$$

$$\text{or, } 1500 \times 10^3 = 1.05 (0.4 \times 25 \times (\pi \times 400^2 - A_{sc}) + 0.67 \times 415 \times A_{sc})$$

on solving

$$A_{sc} = 641.426 \text{ mm}^2$$

$$\text{min area of steel} = 0.8 \times \frac{1}{100} A_g = \frac{0.8 \times \pi \times 400^2}{100} = 1005.309 \text{ mm}^2$$

provide 6  $\times$  16mm  $\phi$  bar.

$$A_{sc, \text{ provided}} = 6 \times 201 = 1206 \text{ mm}^2 > 1005.309 \text{ mm}^2$$

OK

Design of helical bar

$$\text{dia } (\phi) \geq 6 \text{ mm}$$

$$\geq \frac{\phi_{max}}{4} = \frac{16}{4} = 4 \text{ mm}$$

provide 6mm  $\phi$  helical bar.

To satisfy above relation

$$\text{Vol}^m \text{ of spiral reinforcement} = 0.36 \left( \frac{A_g - 1}{A_c} \right) \frac{f_k}{f_y}$$

Vol<sup>m</sup> of core per pitch length

assuming 40mm clear cover.

$$\text{dia of core } (d_c) = 400 - 2 \times 40 = 320 \text{ mm}$$

$$\frac{\pi \times \phi_s^2 \times n \times d_c}{4 \times d_c^2 \times p} \geq 0.36 \left( \frac{D^2}{d_c^2} - 1 \right) \frac{f_k}{f_y}$$

$$\frac{\pi \times d_c^2 \times p}{4}$$

$$\frac{6^2 \times \pi}{320 \times p} \geq 0.36 \left( \frac{400^2}{320^2} - 1 \right) \frac{25}{415} \quad (\text{must satisfy this condition})$$

$$\alpha_s \frac{0.3534}{p} \geq 0.0121$$

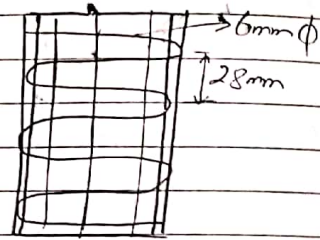
$$p \leq 28.937 \text{ mm}$$

- from code.  $p \geq 25 \text{ mm}$

$$\leq \frac{d}{6} = \frac{320}{6} = 53.33 \text{ mm}$$

$$> 3\phi_{sp} = 3 \times 6 = 18 \text{ mm}$$

So provide 6mm  $\phi$  helical bar at pitch distance 25mm



27) Determine the safe load for short circular column 125mm dia, reinforcement with 6-22mm  $\phi$ . It is provided with 8mm helical bar with pitch of 40mm. Use M20 concrete & Fe 250 steel.

Sol<sup>n</sup>

$$A_{sc} = 6 \times \frac{\pi}{4} \times 22^2 = 2280.796 \text{ mm}^2$$

$$\text{Area of concrete } (A_c) = A_g - A_{sc}$$

$$= \pi \times 125^2 - 2280.796 = 139581.747 \text{ mm}^2$$

Now

$$P_u = 1.05 (1.0 A_c f_{ck} + 0.67 f_y A_{sc})$$

$$= 1.05 \times (1.0 \times 139581.747 + 0.67 \times 250 \times 2280.796) \times 10^{-3}$$

$$= 1573.62 \text{ kN}$$

$$\text{Safe load} = \frac{P_u}{1.05} = \frac{1573.62}{1.05} = 1498.68 \text{ kN}$$

To check the validity of formula

$$\text{Vol}^m \text{ of spiral reinforcement} \geq 0.36 \left( \frac{A_g}{A_c} - 1 \right) f_{ck} \quad d_c = D - 2 \times 1$$

$$\text{Vol}^m \text{ of core per pitch} = \pi \times d_c^2 \times p = \pi \times 120^2 \times 40 = 301440 \text{ mm}^3$$

assuming 40mm clear cover, dia of core ( $d_c$ )

$$= 125 - 2 \times 10$$

$$= 105 \text{ mm}$$

$$= 345 \text{ mm}$$



$$\frac{\pi \phi_{sp}^2 \times n \times d_c}{4} \geq \frac{0.36 (D^2 - 1) \times f_{ck}}{f_y}$$

$$\frac{\pi \cdot d_c^2 \times p}{4}$$

$$0.1 \frac{5^2 \times \pi}{345 \times 4 \times 10} \geq 0.36 \left( \frac{125^2 - 1}{345^2} \right) \times 20$$

$$0.1 \frac{0.0145}{1} \geq 0.0149 \quad \text{OK}$$

Column subjected to axial load and uniaxial moment

Design procedure

- 1) Determine factored axial load and factored bending moment.
- 2) Calculate minimum eccentricity along both axes.
- 3) Min<sup>m</sup> eccentricity multiplied by factored load gives moment due to eccentricity.

Greater of moment due to min<sup>m</sup> eccentricity and factored moment is taken as design bending moment.

- 4) Size of reinforcement, distribution of reinforcement and clear cover is assumed to find  $\frac{d_l}{D}$ ,  $P_u$ ,  $M_u$

$$D \times f_{ck} \times b \times D \quad f_{ck} \times b \times D^2$$

- 5) From graph. find  $P_u$  where  $p = \%$  of steel  $f_{ck}$

- 6) Find the area of longitudinal reinforcement.

- 7) Design of lateral bar.

287 Find the reinforcement required for  $450\text{mm} \times 450\text{mm}$  R.C.C column. subjected for factored load  $250\text{KN}$  and bending moment  $180\text{KNm}$ . Use M25 concrete and Fe 415 steel.

Sol<sup>n</sup>

factored load ( $P_u$ ) =  $250\text{KN}$  (unsupport length)

factored bending moment ( $M_u$ ) =  $180\text{KNm}$  (दिए गये बंधन)

assuming  $20\text{mm}$  rebars at clear.  $40\text{mm}$  distributed equally on four sides.  $e_{min}$  check  $1\% \text{ } \phi$

$$d' = 40 + \frac{20}{2} = 50\text{mm}$$

$$\frac{d'}{d} = \frac{50}{450} = 0.111 \approx 0.1 \text{ (choose nearest value)} \text{ (equivalent rebar)}$$

$P_u = 2500 \times 10^3 = 0.494 \approx 0.493$	0	0	0	0
$f_{ck} \cdot b \cdot d = 25 \times 450 \times 450$	0	1	0	0
	0	1	0	0
$M_u = 180 \times 10^6 = 0.079 \approx 0.08$	0	0	0	0
$f_{ck} \cdot b \cdot d^2 = 25 \times 450 \times 450^2$	0	0	1	0

from graph code pg 37

$$\frac{P}{f_{ck}} = 0.08$$

0.49 in vert. } 0.25% bars  
0.08 in hor. }  $\frac{P}{f_{ck}}$

$$P = 0.08 \times 25 = 2.0$$

$$s, \text{ area of longitudinal bar (A}_{sc}\text{)} = 2\% \text{ of } A_g = \frac{2}{100} \times 450 \times 450$$

$$= 4050\text{mm}^2 \text{ (code no. use not)}$$

provide  $(4 - 25\text{mm} + 8 - 20\text{mm}) \phi$  bar

$$A_{sc} \text{ provided} = 4 \times 490 + 8 \times 314 = 4172\text{mm}^2 > 4050\text{mm}^2$$

297 Design a column subjected to factored axial force and bending moment of  $1000\text{KN}$  and  $100\text{KNm}$  respectively. The unsupported length of column is  $4\text{m}$  with supports rigidly fixed and effectively held in position. Use M20 mix and Fe 500 steel.

Sol<sup>n</sup>

factored load ( $P_u$ ) =  $1000\text{KN}$

ASC =  $2\% \text{ of } A_g$

factored bending moment ( $M_u$ ) =  $100\text{KNm}$

=  $0.02 A_g$

Unsupported length ( $L$ ) =  $4\text{m}$

$A_c = A_g - A_{sc} = 0.98 A_g$

effective length ( $L_e$ ) =  $0.65L$  (code pg 55 table 2E)

$$= 0.65 \times 4 \text{ (case i)}$$

$$= 2.6\text{m}$$

assuming  $e_{min} < 0.05d$  and  $2\% \text{ of steel}$

$$P_u = 0.4 f_{ck} \cdot A_c + 0.67 f_y \cdot A_{sc}$$

$$\therefore 1000 \times 10^3 = 0.4 \times 20 \times 0.98 A_g + 0.67 \times 500 \times 0.02 A_g$$

on solving we get

$$A_g = 68775.79\text{mm}^2$$

assuming square column  $b = d = \sqrt{A_g} = 262.251\text{mm} \approx 300\text{mm}$

provide  $300\text{mm} \times 300\text{mm}$  column

$$e_{min} = \frac{L}{500} + \frac{b}{30} = \frac{4000}{500} + \frac{300}{30} = 18\text{mm} < 20\text{mm}$$

$$0.05d = 0.05 \times 300 = 15\text{mm} < e_{min} \text{ (if } L_{ex} > 20 \text{ then } d \text{ is taken as } 20\text{mm})$$

not OK (if  $L_{ex} > 20$ )

Provide  $500\text{mm} \times 500\text{mm}$  column

$$e_{min} = \frac{4000}{500} + \frac{500}{30} = 21.66 > 20\text{mm}$$

$$0.05d = 0.05 \times 500 = 25\text{mm} > e_{min}$$

OK

$$\frac{L_{ex}}{d} = \frac{2600}{500} = 5.2 < 12 \text{ short column}$$

Assuming 20mm  $\phi$  bar at clear cover 40mm distributed equally on four side,  $d' = \frac{40+20}{2} = 50\text{mm}$

$$\frac{d'}{d} = \frac{50}{500} = 0.1$$

$$P_u = 1000 \times 40^3 = 2 \times 10^{-4} = 0.2$$

for b.d 20x500x500

$$M_u = 10 \times 10^6 = 4 \times 10^{-2} = 0.04$$

for b.d 20x500x500

From graph,

$$\frac{P}{f_{ck}} = 0.01$$

code pg 79

for  $d'/d = 0.1$

$$k_1 k_2 = 0.2, k_1 k_3 = 0.04$$

$$P = 0.01 \times 20 = 0.21 < 2.1$$

So, provided area of steel is sufficient

$$A_{sc} = 2\% \text{ of } A_j = \frac{2}{100} \times 500 \times 500 = 5000 \text{ mm}^2$$

provide 12-25mm  $\phi$  bar  $490 \times 12 = 5880 \text{ mm}^2 > 5000 \text{ mm}^2$

$A_{sc} =$   
OK

for lateral tie bar,

$$\phi \geq 6\text{mm}$$

$$> \frac{\phi_{max}}{4} = \frac{25}{4} = 6.25\text{mm}$$

provide 8mm  $\phi$  bar lateral tie bar.

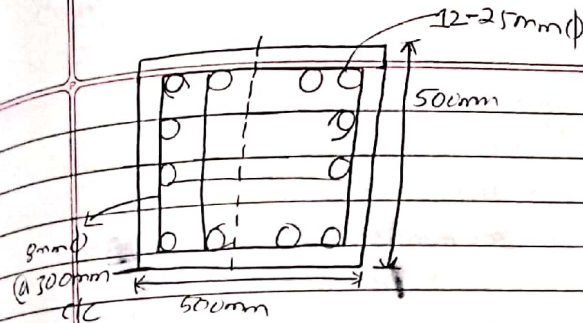
$$\text{Spacing} \leq 300\text{mm}$$

$$< d = 500\text{mm}$$

$$< 16\phi_{min} = 16 \times 25 = 400\text{mm}$$

$$< 48\phi_t = 48 \times 8 = 384\text{mm}$$

provide 8mm  $\phi$  tie bar at spacing 300 c/c.



Column subjected to axial load and biaxial moment.

Design procedure (1% of steel is assumed firstly)

- 1) calculate factored axial load and factored moment along both directions
- 2) calculate minimum eccentricity along both axes.

Minimum eccentricity multiplied by factored load gives moment due to minimum eccentricity.

- 3) design moment is taken as greater value of imposed moment and moment due to minimum eccentricity.

- 4) size of reinforcement, distribution of reinforcement, clear cover and depth of steel is assumed to calculate  $d'$ ,  $P_u$  and  $P$   
 $d$   $f_{ck} \cdot b \cdot d$   $f_{ck}$

- 5) from graph calculate  $M_{ux1}$  and  $M_{uy1}$

- 6) calculate the value of  $x_n$  from given relation,

$$x_n = 0.662 + \frac{1.661 P_u}{P_{u2}}$$

$$\text{where } P_{u2} = 0.45 f_{ck} \cdot A_c + 0.75 f_y A_{sc}$$

- 7) check the relation

$$\left( \frac{M_{ux}}{M_{ux1}} \right)^{\alpha_n} + \left( \frac{M_{uy}}{M_{uy1}} \right)^{\alpha_n} \leq 1$$

- 8) if above relation is true, design is safe otherwise increase the %age of steel and repeat the above procedure

- 9) Design of lateral reinforcement.

307 Design the column for the following data.  
 - factored axial load = 2000 kN, factored bending moment along major axis = 190 kNm, factored bending moment along minor axis = 95 kNm, size of column 300 mm x 500 mm, unsupported length = 3m, use M30 concrete and Fe500 steel.

factored axial load ( $P_u$ ) = 2000 kN  
 factored bending moment along major axis ( $M_{ux}$ ) = 190 kNm  
 factored bending moment along minor axis ( $M_{uy}$ ) = 95 kNm  
 unsupported length ( $L$ ) = 3m

size of column 300 mm x 500 mm  
 Min eccentricity,  $e_{x, min} = \frac{L + d}{500} = \frac{3000 + 500}{500} = 22.667 > 20 \text{ mm}$   
 $e_{y, min} = \frac{L + b}{500} = \frac{3000 + 300}{500} = 22.667 \text{ mm}$

$e_{y, min} = \frac{L + b}{500} = \frac{3000 + 300}{500} = 16 \text{ mm} < 20 \text{ mm}$   
 $= 20 \text{ mm}$

Moment due to minimum eccentricities  $M_{rx} = P_u \cdot e_{x, min}$   
 $= \frac{2000 \times 22.667}{1000} = 45.334 < 190 \text{ kNm}$

$M_{ry} = P_u \cdot e_{y, min} = \frac{2000 \times 20}{1000} = 40 \text{ kNm} < 95 \text{ kNm}$

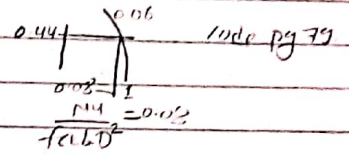
So, design moments are  $M_{ux} = 190 \text{ kNm}$ ,  $M_{uy} = 95 \text{ kNm}$   
 assume 20mm bar distributed equally on four sides at clear cover 40mm and 2% of steel

$d' = \frac{40 + 20}{2} = 50 \text{ mm}$

Moment capacity along major axis

$d'/d = \frac{50}{500} = 0.1$        $\frac{P}{f_{ck} b d} = \frac{2000}{30 \times 300 \times 500} = 0.057$

$P_u = 2000 \times 10^3 = 0.44$   
 $f_{ck} b d = 30 \times 300 \times 500$



from chart  $M_{ux1} = 0.08$   
 $f_{ck} b d^2$

$M_{ux1} = 0.08 \times 30 \times 300 \times 500^2 \times 10^{-6} \text{ kNm}$   
 $= 180 \text{ kNm} < 190 \text{ kNm}$   
 unsafe

So, increase the % of steel to 3.5%  
 moment capacity along major axis  
 $d'/d = 0.1$ ,  $P_u = 0.44$   
 $f_{ck} b d$

$\frac{P}{f_{ck} b d} = \frac{2000}{30 \times 300 \times 500} = 0.116$

from chart  $M_{ux1} = 0.135$  code pg 79  
 $f_{ck} b d^2$

$M_{ux1} = 0.135 \times 30 \times 300 \times 500^2 \times 10^{-6} = 305.75 > 190 \text{ kNm}$   
OK

Moment capacity along minor axis  
 $d'/b = \frac{50}{300} = 0.166 \approx 0.15$

$\frac{P_u}{f_{ck} b d} = 0.44$ ,  $\frac{P}{f_{ck} b d} = 0.116 = 3.5 = 0.116$

from chart  $M_{uy1} = 0.13$  code pg 80  
 $f_{ck} b^2 d$

$M_{uy1} = 0.13 \times 30 \times 300 \times 500 \times 10^{-6} = 175.5 > 95 \text{ kNm}$   
OK

(Ay-Asc)

$$P_{uz} = 0.45 f_c A_c + 0.75 f_y A_{sc}$$

$$= \left[ 0.45 \times 30 \times \left( \frac{500 \times 300 - 3.5 \times 500 \times 300}{100} \right) + 0.75 \times 500 \times \frac{3.5 \times 500 \times 500}{100} \right] \times 10^3$$

$$= 3922.875 \text{ kN}$$

$$\alpha_n = 0.667 + 1.667 \frac{P_u}{P_{uz}}$$

$$= 0.667 + 1.667 \times \frac{2000}{3922.875}$$

$$= 1.513$$

check the relation

$$\left( \frac{M_{ux}}{M_{uxl}} \right)^{\alpha_n} + \left( \frac{M_{uy}}{M_{uyl}} \right)^{\alpha_n} \leq 1$$

$$\left( \frac{140}{303.75} \right)^{1.513} + \left( \frac{95}{175.5} \right)^{1.513} \leq 1$$

$$0.836 < 1$$

So, area of longitudinal reinforcement  $A_{sc} = 3.5\% \text{ of } A_g$

$$= \frac{3.5 \times 300 \times 500 \text{ mm}^2}{100}$$

$$= 5250 \text{ mm}^2$$

provide 12-25mm  $\phi$  bar  $A_{sc}$  provided =  $12 \times 490 = 5880 \text{ mm}^2 > 5250 \text{ mm}^2$

for lateral tie bar  $\phi \geq 6 \text{ mm}$   
 $\geq \phi_{max} = \frac{25}{4} = 6.25 \text{ mm}$

provide 8mm  $\phi$  lateral tie bar.

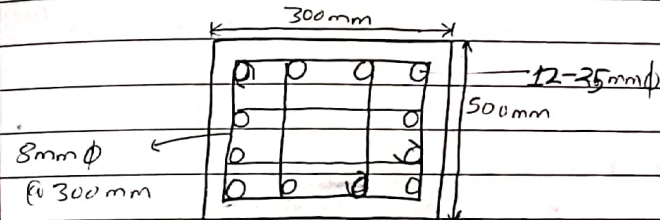
$$\text{spacing} \leq 300 \text{ mm}$$

$$\leq b = 300 \text{ mm}$$

$$\leq 16 \phi_{min} = 16 \times 25 = 400 \text{ mm}$$

$$\leq 48 \phi_t = 48 \times 8 = 384 \text{ mm}$$

So, provide 8mm  $\phi$  lateral tie bar at 300mm c/c



31) A R.C.C carries axial load of 1170 kN accompanied by moment  $M_x = 120 \text{ kNm}$  &  $M_y = 30 \text{ kNm}$  about major & minor axis respectively effective length along major axis = 5.25m & along minor axis = 4m, unsupported length along both axis 4.75m. Design the column section using M20 concrete and Fe415 steel.

Soln

$$P_u = 1170 \times 1.5 = 1755 \text{ kN}$$

$$M_{ux} = 1.5 \times 120 = 180 \text{ kNm}$$

$$M_{uy} = 1.5 \times 30 = 45 \text{ kNm}$$

assume the dimension of column so that it will be short column

$$L_{ex} < 17 \Rightarrow d > L_{ex} = \frac{5250}{12} = 437.5 \text{ mm}$$

$$L_{ey} < 17 \Rightarrow b > L_{ey} = \frac{4000}{12} = 333.33 \text{ mm}$$

Assume  $350 \times 500$  mm column

$$e_{x, \min} = \frac{L}{500} + \frac{d}{30} = \frac{4750}{500} + \frac{500}{30} = 26.167 \text{ mm} > 20 \text{ mm}$$

$$e_{y, \min} = \frac{L}{500} + \frac{d}{30} = \frac{4750}{500} + \frac{350}{30} = 21.167 \text{ mm} > 20 \text{ mm}$$

Moment due to minimum eccentricity  $M_{ax} = P_u \cdot e_{x, \min}$

$$= 1755 \times 26.167 = 45.79 \text{ kNm}$$

$$M_{ay} = P_u \cdot e_{y, \min} = 1755 \times 21.167 = 37.042 \text{ kNm} < 45 \text{ kNm}$$

$$= 37.042 \text{ kNm} < 45 \text{ kNm}$$

So, design moment are  $M_{ux} = 180 \text{ kNm}$

$$M_{uy} = 45 \text{ kNm}$$

Assume 20mm  $\phi$  bars distributed equally on four sides at clear cover 40mm and 2.5% of steel

$$d' = 40 + 20 = 60 \text{ mm}$$

Moment capacity along major axis

$$d'/d = \frac{60}{500} = 0.12$$

$$\frac{P_u}{f_{ck} \cdot b \cdot d} = \frac{1755 \times 10^3}{20 \times 350 \times 500} = 0.501$$

$$\frac{P}{f_{ck}} = \frac{2.5}{20} = 0.125$$

from chart  $M_{ux1} = 0.12$  code pg 75

$$M_{ux1} = 0.12 \times 20 \times 350 \times 500^2 \times 10^{-6} = 210 \text{ kNm} > 180 \text{ kNm}$$

Moment capacity along minor axis

$$d'/b = \frac{60}{350} = 0.171 < 0.15$$

code pg 76

$$\frac{P_u}{f_{ck} \cdot b \cdot d} = 0.501 \quad \frac{P}{f_{ck}} = 0.125$$

from chart  $M_{uy1} = 0.115$

$$f_{ck} \cdot d \cdot b^2$$

$$\alpha_s \cdot M_{uy1} = 0.115 \times 20 \times 500 \times 350^2 \times 10^{-6}$$

$$= 140.875 > 45 \text{ kNm}$$

2.5% of  $A_g$

$$(A_c = A_g - A_{sc})$$

$$P_{u2} = 0.45 f_{ck} \cdot A_c + 0.75 F_y A_{sc}$$

$$= (0.45 \times 20 \times 0.975 \times 350 \times 500 + 0.75 \times 415 \times 0.025 \times 350 \times 500) \times 10^{-3} \text{ kN}$$

$$= 2897.344 \text{ kN}$$

$$x_n = 0.667 + 1.661 \frac{P_{u2}}{P_u}$$

$$= 0.667 + 1.661 \times \frac{1755}{2897.344} = 1.673$$

check for relation

$$\left( \frac{M_{ux}}{M_{ux1}} \right)^{\alpha_n} + \left( \frac{M_{uy}}{M_{uy1}} \right)^{\alpha_n} \leq 1$$

$$0.8 \left( \frac{180}{210} \right)^{1.673} + \left( \frac{45}{140.875} \right)^{1.673} \leq 1$$

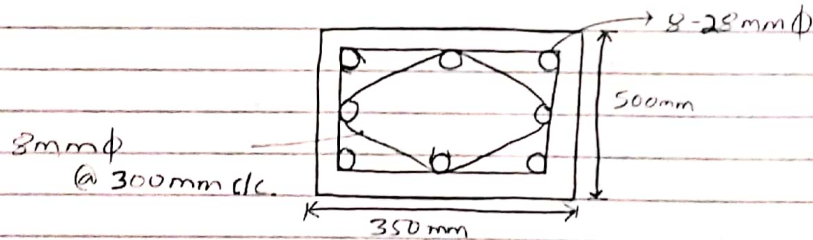
$$0.8 \times 0.92 \leq 1 \quad \underline{OK}$$

So, area of longitudinal bar  $A_{sc} = 2.5\% \text{ of } A_g$   
 $= 0.025 \times 350 \times 500$   
 $= 4375 \text{ mm}^2$

provide 8-25mm  $\phi$  bar  
 for lateral tie bar,  $\phi 26 \text{ mm}$   
 $\Rightarrow d_{max} = \frac{25}{4} = 7 \text{ mm}$

provide 8mm  $\phi$  lateral tie bar.  
 spacing  $< 300 \text{ mm}$   
 $< b = 350 \text{ mm}$   
 $< 16\phi_{min} = 25 \times 16 = 400 \text{ mm}$   
 $< 48\phi_l = 48 \times 8 = 384 \text{ mm}$

provide 8mm  $\phi$  lateral tie bar @ 300mm c/c



Long slender column

slenderness ratio  $(\lambda) = \frac{L_{ex} \text{ or } L_{ey}}{d}$

$\lambda > 12 \Rightarrow$  long slender column

Additional Moment are calculated as

$$M_{ax} = P u_x \cdot e_{ux}$$

$$M_{ay} = P u_y \cdot e_{uy}$$

$$e_{ax} = \frac{d}{2000} \left( \frac{L_{ex}}{d} \right)^2$$

$$e_{ay} = \frac{b}{2000} \left( \frac{L_{ey}}{b} \right)^2$$

Additional moment further reduced by factor  $K$

$$\text{where } K = \frac{P u_x - P u_y}{P u_x - P u_y} \leq 1$$

$$P_{12} = 0.45 P_{cr} A_c + 0.75 f_y A_{sc}$$

$$P_b = \left( \frac{K_1 + K_2}{f_{cr}} \right) \cdot f_{cr} \cdot b \cdot d \Rightarrow K_1 + K_2 = \left( \frac{P_b}{f_{cr} \cdot b \cdot d} \right)$$

where  $p \rightarrow$  % of steel

$K_1, K_2 \rightarrow$  from table 34 (CL-39.7)

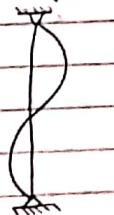
(CL-39.7-1.1 code pg 39)

(for value of  $P_b$  table 60 code pg 94)

Braced column: - If lateral stability to the column as a whole is provided by buttressing wall supports column bent in single curvature



Column bent in double curvature





column bent in single curvature

$$\text{Equivalent moment } M_{ux} = 0.6M_{ux2} + 0.4M_{ux1} \geq 0.4M_{ux2}$$

$$M_{uy} = 0.6M_{uy2} + 0.4M_{uy1} \geq 0.4M_{uy2}$$

$M_{ux2}, M_{uy2}$  = greater end moment.

$M_{ux1}, M_{uy1}$  = smaller end moment.

Total moment,  $M_{tx} = M_{ux} + M_{ax} \geq M_{ux2}$

$$M_{ty} = M_{uy} + M_{ay} \geq M_{uy2}$$

column bent in double curvature

$$\text{Equivalent moment, } M_{ux} = 0.6M_{ux2} - 0.4M_{ux1} \geq 0.4M_{ux2}$$

$$M_{uy} = 0.6M_{uy2} - 0.4M_{uy1} \geq 0.4M_{uy2}$$

Total moment,

$$M_{tx} = M_{ux} + M_{ax} \geq M_{ux2} \quad (\text{code pg 39 notes})$$

$$M_{ty} = M_{uy} + M_{ay} \geq M_{uy2}$$

37) Design the R.C.C column for the following data:-

Size of column 300mm x 400mm

near top ultimate axial load = 1250 kN, ultimate moment at top about

x-axis = 40 kNm, ultimate moment at top about y-axis

15 kNm, ultimate moment at bottom about x & y axis

are 25 kNm & 15 kNm respectively unsupported length can

effective length along x-axis 4.75m & effective length

along y-axis 4.5m use M20 concrete & Fe 415 steel

(assume column bent in single curvature)

(if this condition is not given, assume suitably.)

hints  $M_{ux2} = 40 \text{ kNm}$

$$M_{ux1} = 25 \text{ kNm}$$

$$M_{uy2} = 15 \text{ kNm}$$

$$M_{uy1} = 15 \text{ kNm}$$

$$P_u = 1250 \text{ kN}$$

$$\text{slenderness ratio, } \lambda = \frac{L_{ex}}{d} = \frac{4.75}{0.4} = 11.875 < 12$$

$$= \frac{L_{ey}}{b} = \frac{4.5}{0.3} = 15 > 12$$

so, column is long/slender about minor or y-axis

$$\text{equivalent moment, } M_{ux} = 0.6M_{ux2} + 0.4M_{ux1} \geq 0.4M_{ux2}$$

$$= 0.6 \times 40 + 0.4 \times 25 \geq 0.4 \times 40$$

$$= 34 \geq 16 = 34 \text{ kNm}$$

$$M_{uy} = 0.6M_{uy2} + 0.4M_{uy1} \geq 0.4M_{uy2}$$

$$= 0.6 \times 15 + 0.4 \times 15 \geq 0.4 \times 15$$

$$= 15 \geq 6 = 15 \text{ kNm}$$

$$e_{x, \min} = \frac{L}{500} + \frac{t_d}{30} = \frac{6000}{500} + \frac{400}{30} = 25.33 \text{ mm} > 20 \text{ mm}$$

$$e_{y, \min} = \frac{L}{500} + \frac{b}{30} = \frac{6000}{500} + \frac{300}{30} = 22 > 20 \text{ mm}$$

Moment due to minimum eccentricity

$$M_{ex} = P_u \cdot e_{x, \min} = \frac{1250 \times 25.33}{1000} = 31.662 \text{ kNm} < 34 \text{ kNm}$$

$$M_{ey} = P_u \cdot e_{y, \min} = \frac{1250 \times 22}{1000} = 27.5 \text{ kNm} > 15 \text{ kNm}$$

$$\text{Additional moment } M_{ay} = \frac{k \cdot P_u \cdot b}{2000} \left( \frac{L_{ey}}{b} \right)^2$$

assuming 20mm  $\phi$  bar distributed equally on four sides of 40mm clear cover and 2% of steel,  $d' = \frac{40 + 20}{2} = 50 \text{ mm}$

$$P_{uz} = 0.45 f_{ck} A_c + 0.75 f_{yk} A_{sc}$$

$$= (0.45 \times 20 \times 0.98 + 0.75 \times 415 \times 0.02) \times 300 \times 400 \times 10^{-3} \text{ kN}$$

$$= 1805.4 \text{ kN}$$

$$d'/b = \frac{50}{300} = 0.167$$

$$K_1 = 0.196 + \frac{(0.184 - 0.196) \times (0.167 - 0.15)}{(0.2 - 0.15)} = 0.191$$

$$K_2 = \frac{0.203 + (0.028 - 0.203) \times (0.167 - 0.15)}{(0.2 - 0.15)} = 0.143$$

$$P_b = \left( \frac{K_1 + K_2 P}{f_{ck}} \right) f_{ck} \cdot b \cdot d = \left( \frac{0.191 + 0.143 \times 2}{20} \right) \times 20 \times 300 \times 400 \times 10^{-3} = 492.72 \text{ kN}$$

$$K = \frac{P_{u2} - P_u}{P_{u2} - P_b} = \frac{1805.4 - 1210}{1805.4 - 492.72} = 0.423 < 1$$

$$\text{So, } M_{ay} = 0.423 \times 1250 \times \frac{0.3}{2000} \left( \frac{4500}{300} \right) = 17.845 \text{ kNm}$$

Soed moment

$$M_{tz} = M_{ux} + M_{ax} \geq M_{ux2} \quad (\because M_{ax} = 0 \text{ cause y-axis is fixed})$$

$$= 34 \geq 40 = 40 \text{ kNm}$$

Long column गिर कप्त गिरा

$$M_{ty} = M_{uy} + M_{ay} \geq M_{uy2}$$

$$= 15 + 17.845 \geq 15$$

$$= 45.345 \text{ kNm}$$

Moment capacity along major axis

$$\frac{d'}{l} = \frac{50}{400} = 0.125 < 0.1$$

$$P_u = \frac{1250 \times 10^3}{20 \times 300 \times 400} = 0.521$$

$$\frac{P}{f_{ck}} = \frac{2}{20} = 0.1$$

$$\text{from chart, } \frac{M_{u1}}{f_{ck} \cdot b \cdot d^2} = 0.09$$

$$\text{or, } M_{u1} = 0.09 \times 20 \times 300 \times 400^2 \times 10^{-6}$$

$$= 86.4 \text{ kNm}$$

Moment capacity along minor axis

$$\frac{d'}{b} = 0.167 > 0.15$$

$$\frac{P_u}{f_{ck} \cdot b \cdot d} = 0.521$$

$$P = 0.1$$

$f_{ck}$

$$\text{from chart } \frac{M_{u1}}{f_{ck} \cdot b \cdot d^2} = 0.085$$

$$\text{or } M_{u1} = 0.085 \times 20 \times 400 \times 300^2 \times 10^{-6} = 61.2 \text{ kNm}$$

$$\alpha_n = \frac{0.667 + 1.661 P_u}{P_{u2}} = \frac{0.667 + 1.661 \times 1250}{1805.4} = 1.817$$

check the relation

$$\left( \frac{M_{ux}}{M_{ux1}} \right)^{\alpha_n} + \left( \frac{M_{uy}}{M_{uy1}} \right)^{\alpha_n} \leq 1$$

$$\left( \frac{40}{86.4} \right)^{1.817} + \left( \frac{45.345}{61.2} \right)^{1.817} \leq 1$$

$$0.858 < 1 \quad \text{OK}$$

So, area of longitudinal reinforcement  $A_{sc} = 2.07 A_y$

$$= \frac{2 \times 300 \times 400}{100}$$

$$= 2400 \text{ mm}^2$$

provide 8-20mm  $\phi$  bar longitudinal bar =  $8 \times 314 = 2512 \text{ mm}^2$

$$> 2400 \text{ mm}^2$$

for lateral tie bar  $\phi \geq 6 \text{ mm}$

$$\geq \frac{\phi_{mix}}{4} = \frac{20}{4} = 5 \text{ mm}$$

provide 6mm  $\phi$  tie bar

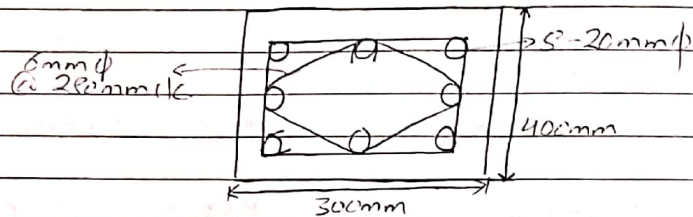
$$\text{spacing} < 300 \text{ mm}$$

$$< b = 300 \text{ mm}$$

$$< 16 \phi_{min} = 16 \times 20 = 320 \text{ mm}$$

$$< 48 \phi_t = 48 \times 6 = 288 \text{ mm}$$

So provide 6mm  $\phi$  lateral tie bar at 280mm c/c spacing



33) Design unbraced rectangular column for following data: Size of column 300mm x 250mm, factored axial load = 750 kN. effective length along x-axis = 3m, effective length along y-axis = 4m, unsupported length = 5m. factored moment in larger dimension = 25 kNm, factored moment in smaller dimension = 15 kNm use M25 concrete & f<sub>y</sub> 500 steel. (d<sub>x</sub>  $\rightarrow$  major)

Sol<sup>n</sup>

Size of column 300mm x 250mm

$$\text{Slenderness ratio } \lambda = \frac{l_{\text{eff}}}{d} = \frac{4000}{300} = 13.33 > 12 \text{ (long)}$$

$$= \frac{l_{\text{eff}}}{b} = \frac{3000}{250} = 12 \text{ (short)}$$

So column is long about major axis.

$$e_{x, \text{min}} = \frac{l}{500} + \frac{d}{30} = \frac{5000}{500} + \frac{300}{30} = 20 \text{ mm}$$

$$e_{y, \text{min}} = \frac{l}{500} + \frac{b}{30} = \frac{5000}{500} + \frac{250}{30} = 18.33 \text{ mm} < 20 \text{ mm}$$

Moment due to minimum eccentricity

$$M_{x, \text{min}} = P_u \cdot e_{x, \text{min}} = \frac{750 \times 20}{1000} = 15 \text{ kNm} < 25 \text{ kNm}$$

$$M_{y, \text{min}} = P_u \cdot e_{y, \text{min}} = \frac{750 \times 20}{1000} = 15 \text{ kNm} < 15 \text{ kNm}$$

$$\text{Additional moment } M_{\text{ax}} = k \cdot P_u \cdot d \left( \frac{d \cdot e_x}{2000} \right)^2$$

assuming 20mm  $\phi$  bar distributed equally on four sides at 40mm clear cover and 2% of steel

$$d' = \frac{40 + 20}{2} = 50 \text{ mm}$$

$$d'/d = \frac{50}{300} = 0.167$$

$$k_1 = \frac{0.196 + (0.184 - 0.196) \cdot (0.167 - 0.15)}{(0.2 - 0.15)} = 0.191$$

$$k_2 = \frac{0.253 + (0.04 - 0.253) \cdot (0.167 - 0.15)}{(0.2 - 0.15)} = 0.183$$

$$P_b = \left( \frac{k_1 + k_2 \cdot P}{f_{ck}} \right) \cdot f_{ck} \cdot b \cdot d = \frac{0.191 + 0.153 \cdot 2}{25} \cdot 25 \cdot 300 \cdot 300 \cdot 10^3 = 385.575 \text{ kN}$$

$$P_{u2} = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$$

$$= (0.45 \cdot 25 \cdot 0.98 + 0.75 \cdot 500 \cdot 0.02) \cdot 300 \cdot 250 \cdot 10^3 = 1389.375 \text{ kN}$$

$$k = \frac{P_{u2} - P_u}{P_{u2} - P_b} = \frac{1389.375 - 750}{1389.375 - 385.575} = 0.637$$

So,

$$M_{\text{ax}} = 0.637 \cdot 750 \cdot 0.3 \cdot (13.33)^2 = 12.738 \text{ kNm}$$

$$\text{final moments, } M_{x, \text{max}} = M_{u, \text{max}} + M_{\text{ax}} = 25 + 12.738 = 37.738 \text{ kNm}$$

$$M_{y, \text{max}} = M_{u, \text{max}} + M_{y, \text{ax}} = 15 \text{ kNm}$$

Moment capacity along major axis

$$d'/h = 0.167 \text{ to } 0.25$$

$$P4 = \frac{750 \times 10^3}{f_{ck} \cdot b \cdot d} = \frac{750 \times 10^3}{25 \times 250 \times 300} = 0.4$$

$$P = \frac{2}{25} = 0.08$$

from chart,  $M_{ux1} = 0.11$   
 $f_{ck} \cdot b \cdot d^2$

$$\text{or, } M_{ux1} = 25 \times 250 \times 300^2 \times 0.11 \times 10^{-6} = 61.875 \text{ kNm}$$

Moment capacity along minor axis

$$d'/b = \frac{50}{250} = 0.2$$

$$P4 = 0.4$$

$$P = 0.08$$

from chart

$$M_{uy1} = 0.1$$

$$\text{or, } M_{uy1} = 25 \times 300 \times 250^2 \times 0.1 \times 10^{-6} = 46.875 \text{ kNm}$$

$$\alpha_n = 0.667 + 1.661 \frac{P4}{P42}$$

$$= 0.667 + 1.661 \times \frac{750}{1389.375}$$

$$= 1.564$$

check relation

$$\left( \frac{M_{ux}}{M_{ux1}} \right)^{\alpha_n} + \left( \frac{M_{uy}}{M_{uy1}} \right)^{\alpha_n} \leq 1$$

$$\text{or, } \left( \frac{37.738}{61.875} \right)^{1.564} + \left( \frac{15}{46.875} \right)^{1.564} \leq 1$$

$$\text{or, } 0.65 \leq 1 \text{ OK}$$

So, area of longitudinal bar  $A_{sc} = 2\% \text{ of } A_g = \frac{2}{100} \times 250 \times 300 = 1500 \text{ mm}^2$

provide 8-16mm  $\phi$  longitudinal bar

fix lateral tie bar.

$$\phi \geq 6 \text{ mm}$$

$$> \frac{\phi_{max}}{4} = \frac{16}{4} = 4 \text{ mm}$$

So, provide 6mm  $\phi$  lateral tie bar.

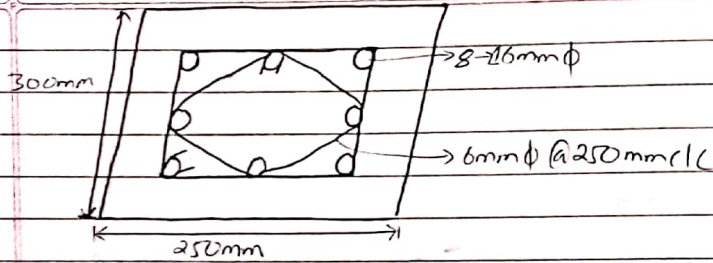
$$\text{Spacing} < 300 \text{ mm}$$

$$< b = 250 \text{ mm}$$

$$\leq 16 \phi_{min} = 16 \times 16 = 256 \text{ mm}$$

$$< 48 \phi_t = 48 \times 6 = 288 \text{ mm}$$

So, provide 6mm  $\phi$  lateral tie bar at 200mm c/c.



34) Design a braced column 300mm $\times$ 400mm in size subjected to following condition.

$$l_e x = l_e y = 6m$$

Factored axial load = 1000 kN.

$M_{ux} = 40$  kNm at top.

$M_{ux} = 30$  kNm at bottom

$M_{uy} = 30$  kNm at top

= 25 kNm at bottom

Use M20 mix and Fe25 steel. Column bent in double curvature.

Sol<sup>n</sup>

Size of column = 300mm $\times$ 400mm

$$\text{Slenderness ratio } (\lambda) = \frac{l_e x}{d} = \frac{6000}{400} = 15 > 12$$

$$\frac{l_e y}{b} = \frac{6000}{300} = 20 > 12$$

So column is long/slender about both axes

Initial moments

$$M_{ux} = 0.6 M_{ux2} - 0.4 M_{ux1} \geq 0.4 M_{ux2}$$

$$= 0.6 \times 40 - 0.4 \times 30 \geq 0.4 \times 40$$

$$= 12 \geq 16 = 16 \text{ kNm}$$

$$M_{uy} = 0.6 M_{uy2} - 0.4 M_{uy1} \geq 0.4 M_{uy2}$$

$$= 0.6 \times 30 - 0.4 \times 25 \geq 0.4 \times 30$$

$$= 8 \geq 12$$

$$= 12 \text{ kNm}$$

Minimum eccentricities

$$e_{x, \min} = \frac{L}{500} + \frac{d}{30} = \frac{6000}{500} + \frac{400}{30} = 25.333 > 20 \text{ mm}$$

$$e_{y, \min} = \frac{L}{500} + \frac{b}{30} = \frac{6000}{500} + \frac{300}{30} = 22 \text{ mm} > 20 \text{ mm}$$

Moment due to minimum eccentricities

$$M_{ex} = P_u \cdot e_{x, \min} = \frac{1000 \times 25.333}{1000} = 25.333 \text{ kNm} > 16 \text{ kNm} \quad (M_{ux})$$

$$M_{ey} = P_u \cdot e_{y, \min} = \frac{1000 \times 22}{1000} = 22 \text{ kNm} > 12 \text{ kNm} \quad (M_{uy})$$

assuming 20mm $\phi$  bars distributed equally on four sides at 40mm clear cover and 3-1.05 steel.

$$d' = \frac{40 + 20}{2} = 50 \text{ mm}$$

$$d'/d = \frac{50}{400} = 0.125$$

$$K_1 = \frac{0.207 + 0.196}{2} = 0.201$$

$$K_2 = \frac{0.328 + 0.203}{2} = 0.265$$

$$P_b = \left( \frac{K_1 + K_2 \cdot P}{f_{ck}} \right) f_{ck} \cdot b \cdot d$$

$$= \left( \frac{0.201 + 0.265 \times 3}{20} \right) \times 20 \times 400 \times 300 \times 10^{-3}$$

$$= 577.8 \text{ kN}$$

$$P_{u2} = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$$

$$= (0.45 \times 20 \times 0.97 + 0.75 \times 415 \times 0.03) \times 300 \times 460 \times 10^{-3}$$

$$= 2168.1 \text{ kN}$$

reduction factor,  $K_y = \frac{P_{u2} - P_u}{P_{u2} - P_b}$

$$= \frac{2168.1 - 1000}{2168.1 - 577.8}$$

$$= 0.735$$

$$= 0.735$$

SO,

$$M_{ax} = K_x \cdot P_u \cdot d \left( \frac{d_c x}{d} \right)^2$$

$$= 0.735 \times 1000 \times 0.4 \frac{(15)^2}{2000}$$

$$= 33.053 \text{ kNm}$$

$$d'/b = \frac{50}{300} = 0.167$$

$$K_1 = \frac{0.196 + (0.184 - 0.196) (0.167 - 0.15)}{(0.2 - 0.15)}$$

$$= 0.192$$

$$K_2 = \frac{0.203 + (0.028 - 0.203) (0.167 - 0.15)}{(0.20 - 0.15)}$$

$$= 0.144$$

$$P_b = \left( \frac{K_1 + K_2 \cdot P}{f_{ck}} \right) f_{ck} \cdot b \cdot d$$

$$= \left( \frac{0.192 + 0.144 \times 3}{20} \right) \times 20 \times 300 \times 400 \times 10^{-3}$$

$$= 512.64 \text{ kN}$$

reduction factor  $k_y = \frac{P_{uz} - P_y}{P_{uz} - P_b}$

$$= \frac{2168.1 - 1000}{2168.1 - 512.64}$$

$\therefore k_y = 0.706$

additional moment  $M_{ay} = k_y \cdot P_u \cdot b \left( \frac{L_{eff}}{b} \right)^2$

$$= 0.706 \times 1000 \times 0.3 \left( \frac{20}{0.3} \right)^2$$

$$= 42.336 \text{ kNm}$$

final moments  $M_{tx} = M_{ux} + M_{ax} \approx M_{ux2}$

$$= M_{ux} + M_{ax} \approx M_{ux2}$$

$$= 25.333 + 33.053 \approx 40$$

$$= 58.386 \text{ kNm}$$

$M_{ty} = M_{uy} + M_{ay} = M_{uy2}$

$$= 12 + 42.336 \approx 30$$

$$= 54.336 \approx 30$$

$$= 54.336 \text{ kNm}$$

Moment capacities along major axis

$\frac{d'}{d} = 0.125 \approx 0.15$

$\frac{P_u}{f_{ck} b d} = \frac{1000 \times 10^3}{20 \times 300 \times 400} = 0.416$

$\frac{P_u}{f_{ck} \cdot 20} = 0.15$

from chart

$M_{ux1} = 0.15$   
 $f_{ck} b d^2$

or  $M_{ux1} = 0.15 \times 20 \times 300 \times 400^2 \times 10^{-6}$

$$= 14416 \text{ Nm}$$

Moment capacity along minor axis

$\frac{d'}{b} = 0.167 \approx 0.15$

$\frac{P_u}{f_{ck} b d} = 0.416$

$\frac{P}{f_{ck}} = 0.15$

from chart  $M_{uy1} = 0.15$   
 $f_{ck} d \cdot b^2$

or,  $M_{uyl} = 0.15 \times 20 \times 400 \times 300^2 \times 10^{-6} \text{ kNm}$

$\therefore M_{uyl} = 108 \text{ kNm}$

$\lambda_n = \frac{0.667 + 1.661 \times p_u}{p_{u2}}$

$= \frac{0.667 + 1.661 \times 1000}{2168.1}$

$= 1.433$

Check the relation

$\left( \frac{M_{ux}}{M_{ux1}} \right)^{\lambda_n} + \left( \frac{M_{uy}}{M_{uy1}} \right)^{\lambda_n} \leq 1$

or,  $\left( \frac{58.386}{144} \right)^{1.433} + \left( \frac{64.336}{108} \right)^{1.433} \leq 1$

or,  $0.752 \leq 1$  OK

So, area of longitudinal bar,  $A_{sc} = 3 \times \text{of } 300 \times 400$

$= \frac{3 \times 300 \times 400}{100}$

$= 3600 \text{ mm}^2$

provide 12-20mm  $\phi$  longitudinal bar  $= 12 \times 314$

$= 3768 \text{ mm}^2 > 3600 \text{ mm}^2$

for lateral tie bar

$\phi = 6 \text{ mm}$

$\geq \frac{\phi_{max}}{4} = \frac{20}{4} = 5 \text{ mm}$

provide 6mm  $\phi$  tie bar.

Spacing  $< 300 \text{ mm}$

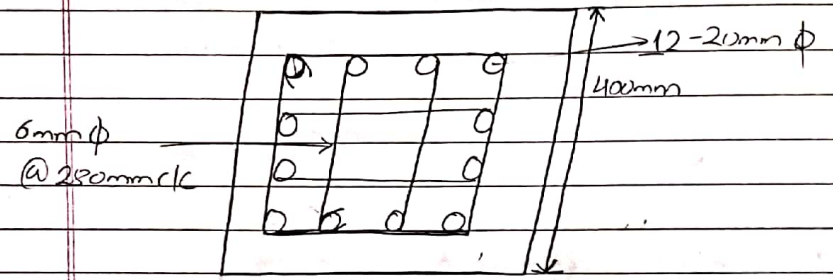
$< b = 300 \text{ mm}$

$< 16\phi_{min} = 16 \times 20 = 320 \text{ mm}$

$< 48\phi_t = 48 \times 6 = 288 \text{ mm}$

So, provide 6mm  $\phi$  lateral tie bar at 280mm c/c spacing





## Design of footing

footing:- footing is a structural element that transfers load from building or individual column to soil underneath. If these loads are to be properly transmitted, footing should be designed to prevent excessive settlement, rotation to avoid differential settlement to provide safety against overturning, sliding etc.

### Types of footing

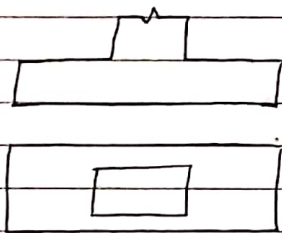
a) shallow footing

If depth of footing is less or equal to width of footing

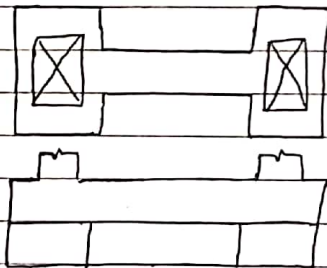
b) deep footing

If depth of footing is greater than width of footing

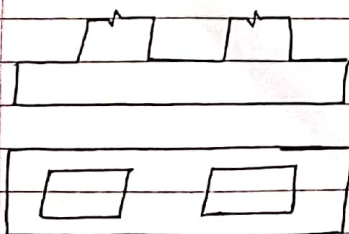
i) Isolated footing



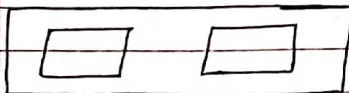
ii) Strapped footing



iii) Combined footing

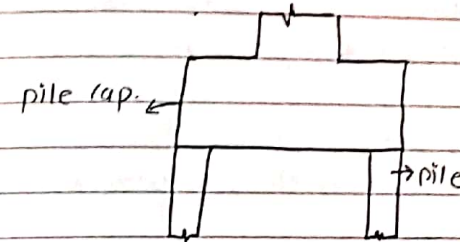
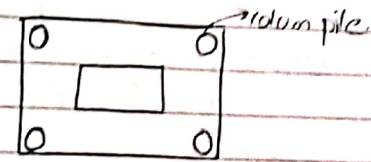


iv) Mat/raft footing :- If soil conditions are poor and differential settlement is to be avoided, mat/raft footing are used.



deep footing

1) pier pile footing



Design of isolated footing

Design procedure

1) calculate the area of footing,  $A_f = \frac{\text{Total load (P)}}{\text{safe bearing capacity of soil (SBCS)}}$

safe bearing capacity of soil (SBCS)

Total load (P) = service load (PS) + self wt. of footing.

If depth of foundation is not given, self wt. of footing is taken as 10% of service load.

If depth of foundation is given, self wt. of footing is taken tentatively equal to the wt. of backfill soil.

2) calculate soil pressure intensity below footing,  $w = \frac{\text{total factored load}}{\text{area of footing}}$

3) calculate max<sup>m</sup> bending moment. critical section for bending moment is taken as faces of column.

footing breadth.

4) calculate depth of footing  $M_{max} = 0.56 f_c k \cdot b \cdot x \cdot (d - 0.42 x)$   
 above calculated depth is increased by (1.5-3) times for shear consideration.   
 ↳ Balanced section assumed

5) calculate area of reinforcement.

$$M_{max} = 0.87 f_y A_{st} \left( d - \frac{f_y A_{st}}{f_{ck} \cdot b} \right)$$

6) check for one way shear.

critical section for one way shear is taken at a distance 'd' i.e. effective depth from faces of column.

7) check for two way shear.

critical section for two way shear is taken at a distance  $\frac{L}{2}$  from faces of column.

8) check for development length.

35) An isolated reinforced concrete footing has to transfer a service load of 800 kN from a square column, 300 mm x 300 mm inside safe bearing capacity of soil is 180 kN/m<sup>2</sup>. Design isolated footing. Use M20 concrete & Fe415 steel.

Soil

Service load (PS) = 800 kN.

safe bearing capacity of soil (SBCS) = 180 kN/m<sup>2</sup>

size of column 300 mm x 300 mm

Total load (P) = service load (PS) + self wt. of footing  
 $= 1.1 \text{ PS} = 1.1 \times 800 = 880 \text{ kN}$ .

area of footing (Af) =  $\frac{\text{total load}}{\text{SBCS}} = \frac{880}{180} = 4.889 \text{ m}^2$

Assuming square footing, side =  $\sqrt{A_f} = \sqrt{4.889} = 2.211 \text{ m}$

provide 2.3 m x 2.3 m square footing

net upward soil pressure (w) =  $\frac{\text{total factored load}}{\text{area of footing}}$

$$= \frac{1.5 \times 880}{2.3 \times 2.3}$$

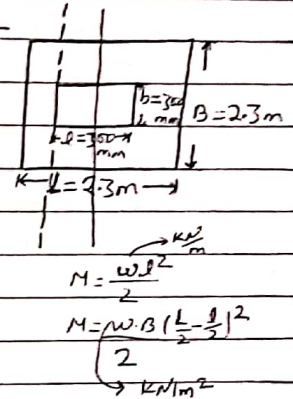
$$= 249.527 \text{ kN/m}^2$$

calculation of bending moment

$$M_{max} = \frac{w B^2}{2} \left( \frac{L-l}{2} \right)^2$$

$$= \frac{249.527 \times 2.3^2}{2} \left( \frac{2.3 - 0.3}{2} \right)^2$$

$$= 286.956 \text{ kNm}$$



$$M = \frac{w l^2}{2}$$

$$M = w \cdot B \left( \frac{L-l}{2} \right)^2$$

kN/m<sup>2</sup>

calculate depth of footing → breadth of footing = 2.3 m

$$M_{max} = 0.36 f_{ck} \cdot b \cdot x_u (d - 0.42 x_u)$$

$$\text{or, } 286.956 \times 10^6 = 0.36 \times 20 \times 2300 \times 0.48 d (d - 0.42 \times 0.48 d)$$

on solving we get,

$$d = 212.640 \text{ mm}$$

(effective depth of footing)

increase the depth of footing by 25 times

$$d = 2.5 \times 212.640 = 531.602 \approx 540 \text{ mm}$$

providing 60 mm effective cover, overall depth (D) = 540 + 60 = 600 mm

calculate the area of reinforcement

$$M_{max} = 0.87 f_y A_{st} \left( d - \frac{f_y A_{st}}{f_{ck} \cdot b} \right)$$

$$\text{or, } 286.956 = 0.87 \times 415 \times A_{st} \left( 540 - \frac{415 \times A_{st}}{20 \times 2300} \right)$$

\*10<sup>6</sup>

on solving we get

$$A_{st} = 1507.907 \text{ mm}^2$$

$$\text{provide } 12 \text{ mm } \phi \text{ at spacing} = \frac{\pi \times 12^2}{4} \times 2300 = 172.278 \approx 170 \text{ mm}$$

$$\frac{1507.907}{170} \times 2300$$

provide 12mm  $\phi$  bar @ 70 mm c/c.

$$\text{Actual } A_{st, \text{ provided}} = \frac{\pi \times 12^2}{4} \times 2300 \text{ mm}^2 = 1530.140 \text{ mm}^2$$

$$\frac{1530.140}{170}$$

check for one way shear.

$$\text{Shear force } (V_{max}) = wB \left( \frac{L-l-d}{2} \right)$$

$$= 249.527 \times 2.3 \times$$

$$\left( \frac{2.3 - 0.3 - 0.54}{2} \right)$$

$$= 263.999 \approx 264 \text{ kN.}$$

$$\text{Ultimate shear stress } (T_u) = \frac{V_{max}}{Bd} = \frac{264 \times 10^3}{2300 \times 540} = 0.213 \text{ N/mm}^2$$

$$\% \text{ of steel} = \frac{A_{st} \times 100}{B \cdot d} = \frac{1530.140 \times 100}{2300 \times 540} = 0.123 \% < 0.15$$

From code p. 940

$$T_c = 0.28 \text{ N/mm}^2$$

$$T_c' = K \cdot T_c \text{ where } K=1 \text{ for } D > 300 \text{ mm}$$

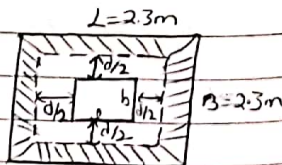
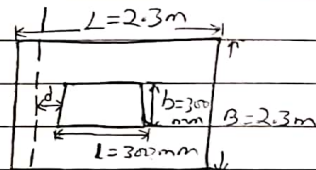
$$= 1 \times 0.28 = 0.28 \text{ N/mm}^2 > 0.213 \text{ N/mm}^2 \text{ OK}$$

check for two way shear.

$$\text{Shear force } (V_{max})$$

$$= w \times [L \times B - (l+d)(b+d)]$$

$$= 249.527 \times (2.3 \times 2.3 - (0.3 + 0.54) \times (0.3 + 0.54)) = 1143.932 \text{ kN}$$



$$\text{ultimate shear } (T_u) = V_{max} \text{ stress} \quad \text{whole perimeter} \rightarrow \text{length } (p_o)$$

$$2 \times \frac{(l+d+b+d)}{2} \times \text{downside depth}$$

$$= 1143.932 \times 10^3$$

$$2 \times (300 + 540 + 300 + 540) \times 540$$

$$= 0.63 \text{ N/mm}^2$$

now,

$$T_c' = K_s \cdot T_c$$

$$\text{where } K_s = 1 + \beta \leq 1$$

$$\beta = \frac{\text{shorter side}}{\text{longer side}} = 1$$

$$K_s = 1 + 1 \leq 1$$

$$= 1$$

$$T_c = 0.25 \sqrt{f_{ck}} = 0.25 \sqrt{20} = 1.118 \text{ N/mm}^2 > 0.63 \text{ N/mm}^2 \text{ OK}$$

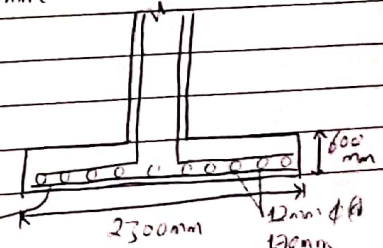
check for development length

$$(l_d)_{\text{required}} = \frac{0.87 f_y \phi}{4 T_c} = \frac{0.87 \times 415 \phi}{4 \times 1.118} = 136$$

$$\text{for } 12 \text{ mm } \phi \text{ bar, } (l_d)_{\text{required}} = 47 \times 12 = 564$$

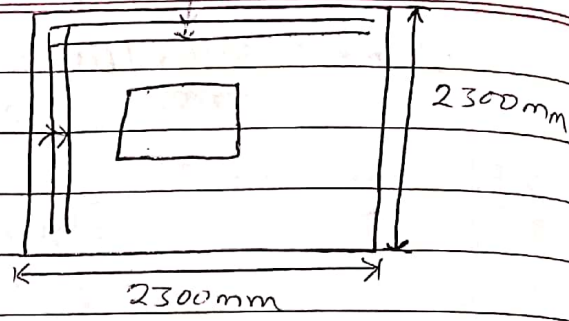
$$(l_d)_{\text{available}} = \frac{L-l}{2} - (\text{bar curv}) = \frac{2300-300}{2} - 42 = 950 \text{ mm}$$

$$950 \text{ mm} > 564 \text{ mm} \text{ OK}$$



1.2mm  $\phi$   
@ 170mm

1.2mm  $\phi$   
@ 170mm



(Unit wt. of soil is  $18 \text{ kN/m}^3$ )

36) Design a rectangular isolated footing for column  $400 \text{ mm} \times 600 \text{ mm}$  provided with 8-25 mm dia longitudinal bar carries a service load  $3500 \text{ kN}$  assume safe bearing capacity of  $175 \text{ kN/m}^2$  at a depth of  $1.8 \text{ m}$  below ground level. Use M20 concrete and Fe415 steel.

sol<sup>n</sup>

Size of column =  $400 \text{ mm} \times 600 \text{ mm}$

Safe bearing capacity of soil (SBCS) =  $175 \text{ kN/m}^2$

Service load (PS) =  $3500 \text{ kN}$ .

Total load (P) = service load (PS) + self wt of footing

$$= PS + \gamma \times \rho \times f \times PS$$

SBCS

$$= 3500 + 18 \times 1.8 \times 3500 / 175$$

$$= 4148 \text{ kN}$$

Area of footing = total load (P)

safe bearing capacity of soil (SBCS)

$$= \frac{3500 + 18 \times 1.8 \times 3500}{175}$$

$$= \frac{4148}{175} \quad (\because \text{no factoring})$$

$$= 23.703 \text{ m}^2$$

ratio of length and breadth of footing is kept same to that of column

$$\frac{L}{B} = \frac{l}{b} = \frac{600}{400} = 1.5$$

↓  
footing column

$$\therefore L = 1.5B$$

Depth of footing  
दिए गयी है।  
यदि  $1.8 \text{ m}$  गहराई  
है तो  $1.8 \times 18 \times 3500$

now, Area of footing ( $A_f$ ) =  $L \times R = 1.5R \times R = 23.703$

$$or, R = \frac{\sqrt{23.703}}{1.5}$$

$$= 3.935 \approx 4m$$

$$L = 1.5R = 1.5 \times 4 = 6m$$

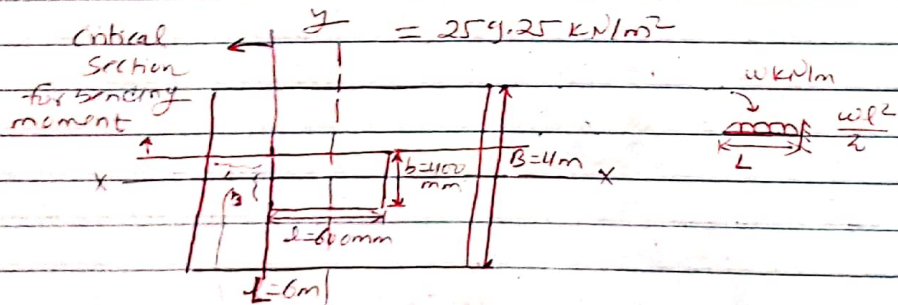
So, provide  $4m \times 6m$  footing.

Net upward soil pressure ( $w$ ) = factored total load.

$$\frac{4 \times 6}{\text{Area of footing}}$$

$$= \frac{1.5 \times 4118}{4 \times 6}$$

$$= 259.25 \text{ kN/m}^2$$

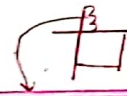


Calculation of Bending Moment.

B.M. along Y-Y axis

$$M_{x-x} = \frac{w \cdot B \cdot (L/2 - l/2)^2}{2}$$

$$= \frac{259.25 \times 4 \cdot (6/2 - 0.6/2)^2}{2} = 3779.865 \text{ kNm}$$



Bending Moment along X-X axis

$$M_{x-x} = \frac{w \cdot L \cdot (B/2 - b/2)^2}{2}$$

$$= \frac{259.25 \times 6 \cdot (4/2 - 0.4/2)^2}{2}$$

$$= 2519.91 \text{ kNm}$$

$M_{y-y}$  is greater than  $M_{x-x}$

Calculation of depth of footing

$$M_{y-y} = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

$$or, 3779.865 = 0.36 \times 20 \times 4000 \times 0.415 \cdot d \cdot (d - 0.42 \times 0.48d)$$

( $b = l$  for  $M_{x-x}$ )  
( $M_{x-x}$  is greater than  $M_{y-y}$ )

on solving we get,

$$d = 585.209 \text{ mm}$$

increase the depth by 2 times =  $585.209 \times 2 = 1170.418 \approx 1175 \text{ mm}$

providing 60mm effective cover, overall depth ( $D$ ) =  $1175 + 60$

$$= 1235 \text{ mm}$$

Calculation of area of reinforcement

along Y-Y direction

$$M_{y-y} = 0.87 f_y A_{stj} (d - f_y A_{stj} / f_{ck} b)$$

$$or, 3779.865 = 0.87 \times 415 \times (1175 - 415 \cdot A_{stj} / (20 \times 4000)) \times A_{stj}$$

on solving we get,

$$A_{stj} = 9290.966 \text{ mm}^2$$

provide 16mm  $\phi$  bar at spacing =  $\frac{\pi \times 16^2}{4} \times 1000 = 86.56 \text{ mm}$

$$\frac{9290.966}{86.56} = 107.33 \approx 108 \text{ mm}$$

(x-axis is parallel along y-direction) (bar spacing)

provide 16 mm  $\phi$  bar @ 80 mm c/c

$$\text{actual } (A_{st}) \text{ provided} = \frac{\pi \times 16^2}{80} \times 4000 \text{ mm}^2$$

$$= 10053.096 \text{ mm}^2$$

Reinforcement along x-x dir<sup>n</sup>.

$$M_{x-x} = 0.87 f_y A_{st} x \left( d - \frac{f_y A_{st} x}{f_{ck} L} \right)$$

$$\text{or } 2599.91 \times 10^6 = 0.87 \times 415 \times A_{st} x \left( 1175 - \frac{415 \cdot A_{st} x}{20 \times 6000} \right)$$

on solving we get

$$A_{st} x = 6047.553 \text{ mm}^2$$

$$\text{provide } 12 \text{ mm } \phi \text{ at spacing} = \frac{\pi \times 12^2}{4} \times 6000$$

$$6047.553$$

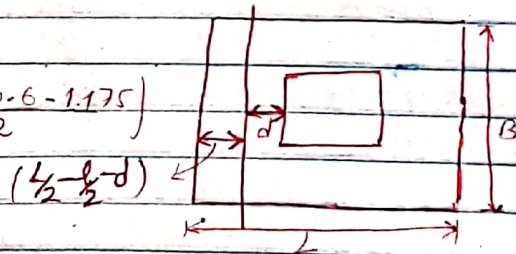
$$= 112.208 \approx 110 \text{ mm}$$

check for one way shear.

$$V_{y-y} = w \cdot B \left( \frac{L}{2} - \frac{l}{2} - d \right)$$

$$= 259.25 \times 4 \left( \frac{6}{2} - \frac{0.6}{2} - 1.175 \right)$$

$$= 1581.425 \text{ kN}$$



$$V_{x-x} = w \cdot L \left( \frac{B}{2} - \frac{b}{2} - d \right) = 259.25 \times 6 \left( \frac{4}{2} - \frac{0.4}{2} - 1.175 \right)$$

$$= 972.1875 \text{ kN}$$

$$V_{y-y} > V_{x-x}$$

$$\text{Ultimate shear stress} = \frac{V_{y-y}}{B \cdot d} = \frac{1581.425}{4000 \times 1175} = 1581.425 \times 10^3$$

$$= 0.336 \text{ N/mm}^2$$

$$\% \text{ age of steel} = \frac{A_{st}}{B \cdot d} \times 100\%$$

$$= \frac{10053.096}{4000 \times 1175} \times 100\%$$

$$= 0.214\%$$

M20

Now from code pg 40

$$0.15 \rightarrow 0.28$$

$$0.25 \rightarrow 0.36$$

$$\tau_c = 0.2149 = 0.331$$

$$\text{design shear stress } (\tau_c') = K \cdot \tau_c \text{ where } K \text{ for } \phi > 300 \text{ mm}$$

$$= 1 \times 0.331 = 0.331 \approx \tau_u = 0.336 \text{ OK}$$

$$\text{N/mm}^2$$

Value of K code pg 84 cl. 40.2

If unsafe increased by 3 or 4

times d. i.e. d = 3 × 585.209

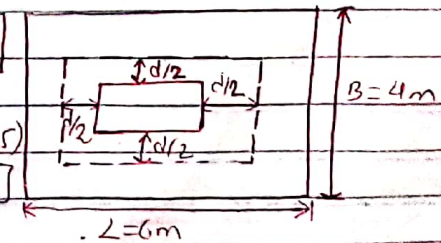
Or do truncate the spacing

check for two way shear.

$$\text{shear force } (V) = w [L \cdot B - (l + d)(b + d)]$$

$$= 259.25 (6 \times 4 - (0.6 + 1.175) \times (0.4 + 1.175))$$

$$= 5497.234 \text{ kN}$$





Ultimate shear stress ( $T_c$ ) =  $V' = \frac{V}{b \cdot d}$

$$= \frac{259.25 \times 10^3}{2 \times (600 + 1175 + 400 + 1175) \times 1175}$$

$$= 0.0329 \text{ N/mm}^2$$

design shear stress ( $T_c'$ ) =  $K_s \cdot T_c$   
 where  $K_s = (0.5 + \beta) \leq 1$

$\beta = \frac{\text{shorter side}}{\text{longer side}} = \frac{400}{600} = 0.667$

$$K_s = 0.5 + 0.667 < 1$$

$$1.667 \leq 1 - 1$$

$$T_c = 0.25 \sqrt{f_{ck}} = 0.25 \sqrt{20} = 1.118$$

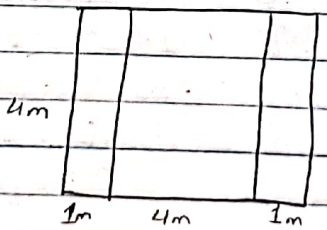
$$T_c' = 1 \times 1.118 = 1.118 \text{ N/mm}^2 > 0.0329 \text{ N/mm}^2 \text{ OK}$$

reinforcement in central band = 2 \* reinforcement in shorter dir<sup>n</sup>  
 $\beta \cdot l$

$\beta = \frac{\text{longer side}}{\text{shorter side}} = \frac{600}{400} = 1.5$

$$= \frac{2 \times 10053.076}{1.5 \cdot 1}$$

$$= 8042.476 \text{ mm}^2$$



provide 16mm  $\phi$  bar at spacing equals  $\frac{\pi \cdot 16^2}{4} \times 4000$   
 $= 100 \text{ mm}$

check for development length,  $(l_{d})_{\text{required}} = \frac{0.87 f_y \phi}{4 \tau_{bd}}$

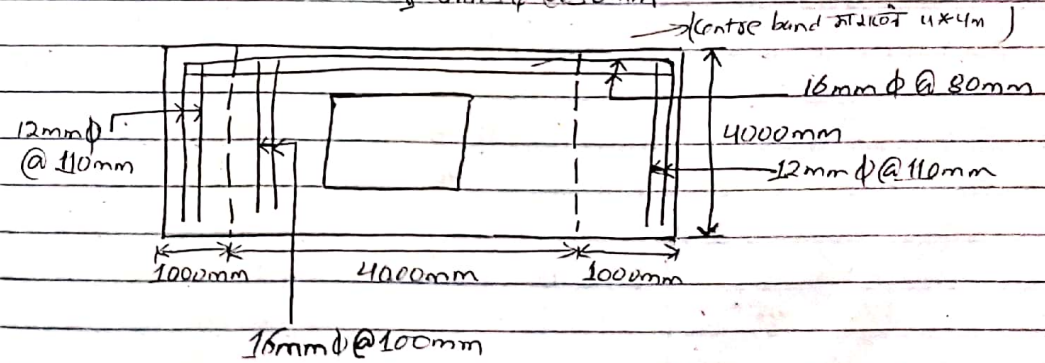
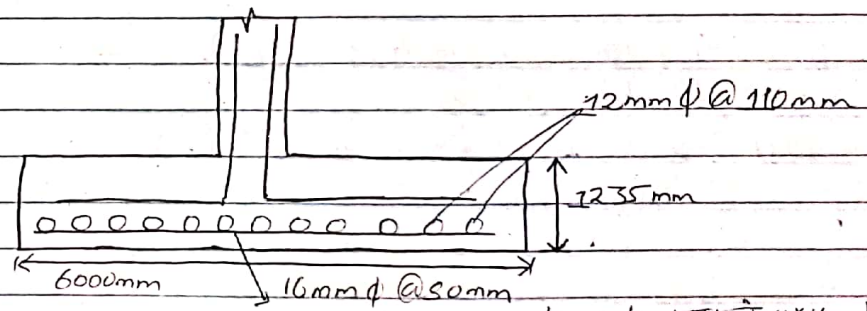
$$= \frac{0.87 \times 415 \phi}{4 \times 1.2 \times 1.6}$$

$$= 47 \phi$$

$$= 47 \times 16 = 752$$

$(l_{d})_{\text{available}} = \frac{L - \phi}{2} - \text{clear cover}$

$$= \frac{6000 - 600 - 50}{2} = 2650 \text{ mm} > 752 \text{ mm} \text{ OK}$$



37) Design a footing to support a 300mm x 400mm column. The column carries factored axial load of 1400 kN and a factored moment of 90 kNm. The safe soil pressure is 200 kN/m<sup>2</sup> at a 1.5m depth. column is reinforced with 6-22mm  $\phi$  bar. Unit wt. of soil is 20 kN/m<sup>3</sup>. Use M20 concrete and Fe415 steel.

sol<sup>n</sup> (Moment is given by axis/dim is not specified)

- factored axial load (P<sub>u</sub>) = 1400 kN
- factored B.M. (M<sub>u</sub>) = 90 kNm
- safe bearing capacity of soil (SBC) = 200 kN/m<sup>2</sup>

Service load (P<sub>s</sub>) =  $\frac{P_u}{1.5} = \frac{1400}{1.5} = 933.333 \text{ kN}$

total load (P) = service load (P<sub>s</sub>) + self wt. of footing

$$= 933.333 + \frac{\gamma \cdot D \cdot P_s}{\text{SBC}}$$

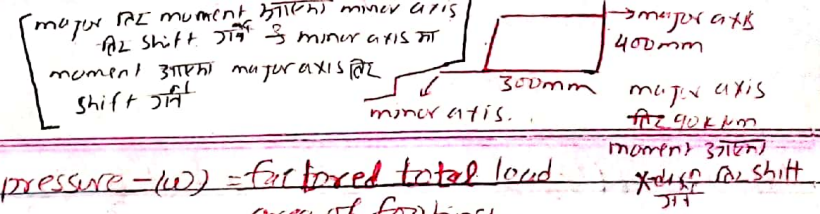
$$= 933.333 + \frac{20 \times 1.5 \times 933.333}{200}$$

$$= 1073.533 \text{ kN}$$

area of footing (A<sub>f</sub>) =  $\frac{\text{Total load (P)}}{\text{SBC}} = \frac{1073.533}{200} = 5.367 \text{ m}^2$

assume square footing, side =  $\sqrt{A_f} = \sqrt{5.367} = 2.317 \approx 2.5 \text{ m}$

provide 2.5m x 2.5m square footing.

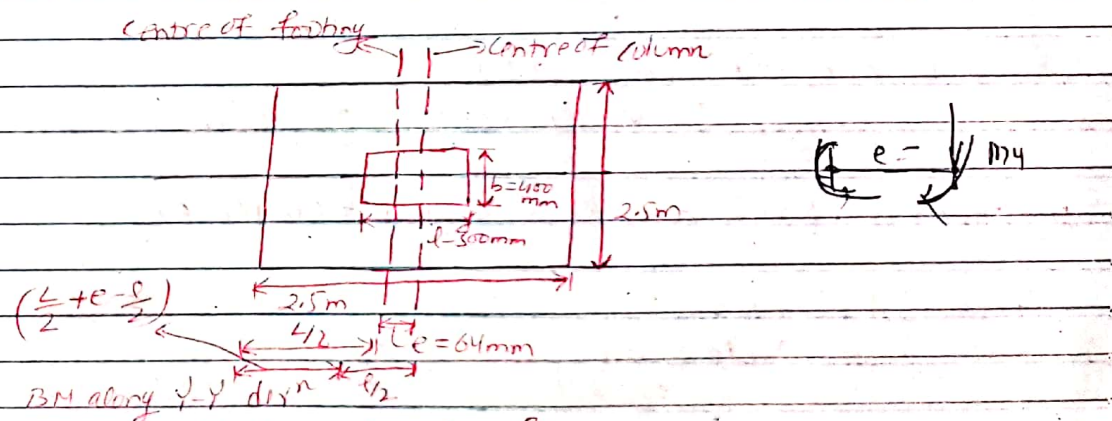


Net upward soil pressure - (w) =  $\frac{\text{factored total load}}{\text{area of footing}}$

$$= \frac{1073.533 \times 1.5}{2.5 \times 2.5}$$

$$= 257.599 \approx 257.6 \text{ kN/m}^2$$

To compensate effect of B.M., shift the axis of footing from axis of column by eccentricity (e) =  $\frac{M_u}{P_u} = \frac{90 \times 10^3}{1400} = 64.286 \text{ mm} \approx 64 \text{ mm}$



BM along y-y axis

$$M_y - y = w \cdot B \left( \frac{L}{2} - \frac{d}{2} + e \right)^2$$

$$= \frac{257.6 \times 2.5 \times \left( \frac{2.5 - 0.3 + 0.064}{2} \right)^2}{2}$$

$$= 436.271 \text{ kNm}$$

$$M_x-x = w \cdot L \left( \frac{B}{2} - \frac{b}{2} \right)^2 = 257.6 \times 2.5 \left( \frac{2.5 - 0.4}{2} \right)^2$$

$$= 355 \text{ kNm}$$

calculation of depth of footing

$$M_y-y = 0.36 \times f_{ck} \cdot b \cdot x_u (d - 0.42x_u)$$

$$\text{or } 436.22 \times 10^6 = 0.36 \times 20 \times 2500 \times 0.42x_u (d - 0.42 \times 0.48d)$$

on solving we get

$$d = 257.486 \text{ mm}$$

increase the depth of footing atleast 2 times

$$d = 2 \times 257.486 = 507.972 \approx 510 \text{ mm}$$

provide 60mm effective cover overall depth (D) = 510 + 60 = 570mm

calculation of reinforcement

$$M_y-y = 0.87 f_y A_{st} \left( \frac{d - f_y A_{st} x}{f_{ck} b} \right)$$

$$\text{or } 436.22 \times 10^6 = 0.87 \times 415 \times A_{st} \left( \frac{510 - 415 A_{st} x}{20 \times 2500} \right)$$

on solving we get

$$A_{st} = 2465.488 \text{ mm}^2$$

$$\text{provide } 12 \text{ mm } \phi \text{ bar at spacing} = \frac{\pi \times 12^2}{4} \times 2500$$

$$= 248488$$

$$= 114.541 \text{ mm}$$

$$= 100 \text{ mm}$$

provide 12mm  $\phi$  bar @ 100mm c/c

$$\text{Actual } A_{st} \text{ provided} = \frac{\pi \times 12^2}{4} \times 2500 \text{ mm}^2$$

$$= 2822.033 \text{ mm}^2$$

$$M_x-x = 0.87 f_y A_{st} x \left( \frac{d - f_y A_{st} x}{f_{ck} L} \right)$$

$$\text{or } 355 \times 10^6 = 0.87 \times 415 \times A_{st} x \left( \frac{510 - 415 \times A_{st} x}{20 \times 2500} \right)$$

on solving we get

$$A_{st} x = 1992.541 \text{ mm}^2$$

$$\text{provide } 12 \text{ mm } \phi \text{ bar at spacing} = \frac{\pi \times 12^2}{4} \times 2500$$

$$= 141.9$$

$$= 140 \text{ mm}$$

check for one way shear

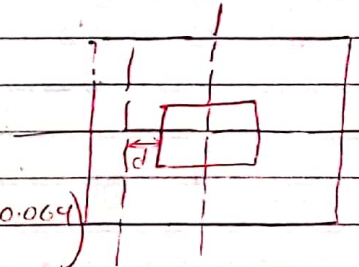
$$V_x-x = w \cdot B \left( \frac{L}{2} - \frac{d}{2} + e \right)$$

$$= 257.6 \times 2.5 \left( \frac{2.5 - 0.3 - 0.510 + 0.064}{2} \right)$$

$$= 426.776 \text{ KN}$$

$$V_x-x = w \cdot L \left( \frac{B}{2} - \frac{b}{2} - d \right)$$

$$= 257.6 \times 2.5 \left( \frac{2.5 - 0.4 - 0.510}{2} \right) = 347.76$$



$$\text{Ultimate shear stress } (T_u) = \frac{V_u}{b \cdot d} = \frac{421.176 \times 10^3}{2500 \times 510} = 0.33 \text{ N/mm}^2$$

$$\% \text{ of steel} = \frac{A_{st}}{b \cdot d} \times 100\%$$

$$= \frac{2827.433}{2500 \times 510} \times 100$$

$$= 0.222$$

from code pg 40

$$0.15 \rightarrow 0.28$$

$$0.25 \rightarrow 0.36$$

$$T_u = 0.222 \times 1.5 = 0.3336 \text{ N/mm}^2$$

$$T_c' = K \times T_c \text{ where } K = 1.6 \text{ for } \phi > 300 \text{ mm (code pg 39 4.2.2.1)}$$

$$= 1 \times 0.333 = 0.333 \text{ N/mm}^2 > 0.333 \text{ N/mm}^2$$

OK

check for two way shear.

$$V = w (L \times B - (l + d) / (b + d))$$

$$= 257.6 (2.5 \times 2.5 - (0.3 + 0.51) / (0.4 + 0.51))$$

$$= 1420.123 \text{ kN}$$

$$\text{Ultimate shear } (T_u) = \frac{V}{b \cdot d} = \frac{1420.123 \times 10^3}{2 \times (800 + 510 + 400 + 510) \times 510}$$

$$= 0.8094 \text{ N/mm}^2$$

$$T_c' = K_s T_c$$

$$K_s = 0.5 + \beta \leq 1$$

$$\beta = \frac{\text{shorter side (column)}}{\text{longer side}} = \frac{300}{400} = 0.75$$

$$K_s = 0.5 + 0.75 = 1.25 \leq 1 = 1$$

$$T_c = 0.25 \sqrt{f_{ck}} = 0.25 \sqrt{20} = 1.118 \text{ N/mm}^2$$

$$T_c' = 1 \times 1.118 = 1.118 \text{ N/mm}^2 > 0.8094 \text{ N/mm}^2$$

OK

reinforcement in central band =  $\frac{2}{\beta + 1}$  × reinforcement in shorter dir<sup>n</sup>

$$\beta = \frac{\text{longer side (footing)}}{\text{shorter side (footing)}} = \frac{2.5}{2.5} = 1$$

$$= \frac{2}{1+1} \times A_{st} = \frac{2}{2} \times 2827.433 = 2827.433 \text{ mm}^2$$

So, provide 12 mm  $\phi$  bar @ 100 mm

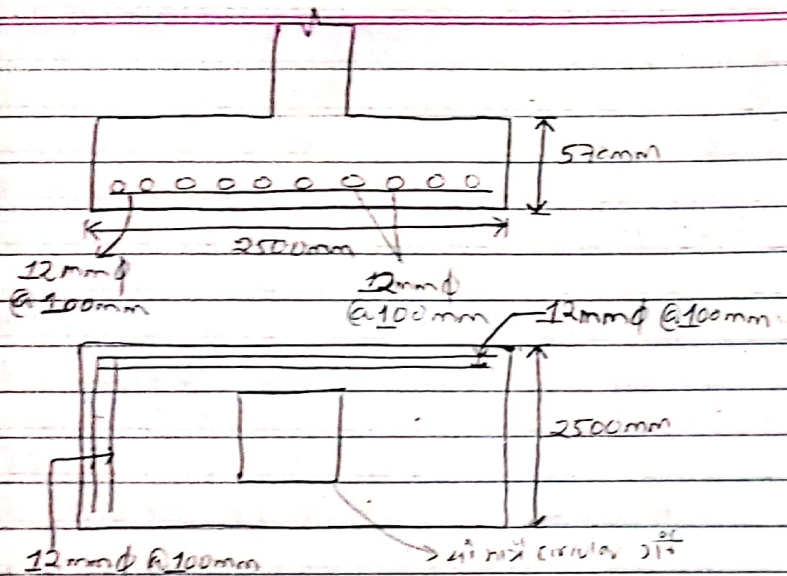
check for development length.

$$(l_d)_{\text{required}} = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 415 \phi}{4 \times 1.2 \times 1.6} = 47 \times 12 = 564 \text{ mm}$$

$$(l_d)_{\text{available}} = \frac{L}{2} - \frac{l}{2} - e - \text{clear cover}$$

$$= \frac{2500}{2} - \frac{300}{2} - 64 - 50 = 986 \text{ mm} > 564 \text{ mm}$$

OK



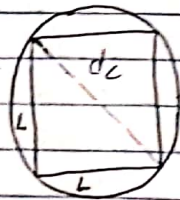
NOTE:-

If circular column is given,  
circular column is converted to equivalent square column  
of side  $d_c$  where  $d_c = \text{dia of circular column}$   
 $\sqrt{2}$

$$L^2 + L^2 = d_c^2$$

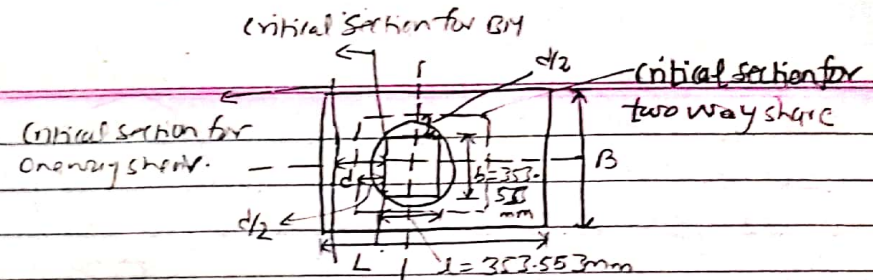
$$2L^2 = d_c^2$$

$$\therefore L = \frac{d_c}{\sqrt{2}}$$



If dia. of circular column is 500 mm

$$\text{side of square column} = \frac{500}{\sqrt{2}} = 353.553 \text{ mm}$$



387 Design an isolated footing which has 400 mm dia of column with 8-20 mm dia longitudinal bar and carrying a load 800 kN. Assume soil with safe bearing capacity of 80 kN/m<sup>2</sup> at a depth 2 m below ground. Use M20 grade concrete for column & M20 concrete for footing and Fe25 steel. Unit wt of soil 12 kN/m<sup>3</sup>.

Service load (PS) = 800 kN

Safe bearing capacity of soil (SBCS) = 80 kN/m<sup>2</sup>

Depth of footing (DF) = 2 m

Total load (P) = Service load (PS) + self wt. of

footing

$$= 800 + \gamma \cdot DF \cdot PS$$

$$= 800 + 12 \times 2 \times 800$$

$$= 1160 \text{ kN}$$

Area of footing = total load

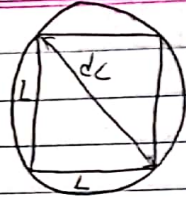
$$\text{safe bearing capacity of soil (SBCS)}$$

$$= \frac{1160}{80} = 14.5 \text{ m}^2$$

Here

$$L^2 + L^2 = d_c^2$$

$$L = \frac{d_c}{\sqrt{2}} = \frac{400}{\sqrt{2}} = 282.842 \text{ mm}$$



converting circular column to equivalent square column.

now

$$\text{Area of footing } (A_f) = L \times L = 14.5$$

$$L = \sqrt{14.5} = 3.8124 \text{ m}$$

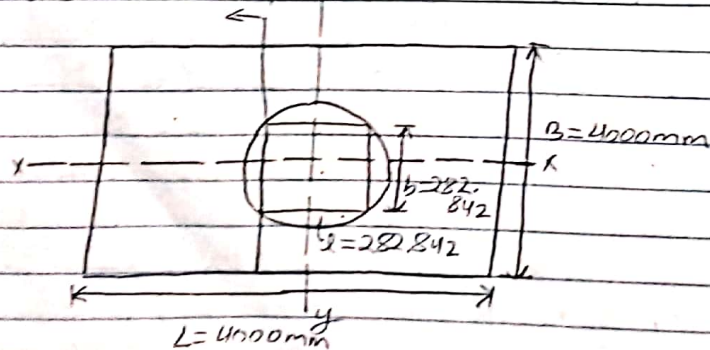
so, provide  $4 \times 4$  (m) footing

net upward soil pressure ( $w$ ) =  $\frac{\text{factored total load}}{\text{area of footing}}$

$$= \frac{1.5 \times 1160}{4 \times 4}$$

$$= 108.75 \text{ kN/m}^2$$

Critical section for B.M.



calculation of Bending Moment.

B.M along Y-Y axis.

$$M_{y-y} = \frac{w \cdot B \left( \frac{L}{2} - \frac{d}{2} \right)^2}{2}$$

$$= \frac{108.75 \times 4 \times \left( \frac{4}{2} - \frac{0.282842}{2} \right)^2}{2}$$

$$= 751.3137 \text{ kNm}$$

Bending Moment along X-X axis

$$M_{x-x} = \frac{w \cdot L \left( \frac{B}{2} - \frac{d}{2} \right)^2}{2}$$

$$= \frac{108.75 \times 4 \times \left( \frac{4}{2} - \frac{0.282842}{2} \right)^2}{2}$$

$$= 751.3137 \text{ kNm} \quad M_{yy} = M_{xx}$$

calculation of depth of footing

$$M_{y-y} = 0.36 f_c k \cdot b \cdot x_v (d - 0.42 x_v)$$

$$\text{or, } 751.3137 \times 10^6 = 0.36 \times 20 \times 4000 \times 0.48 d \times (d - 0.42 \times 0.48 d)$$

on solving, we get,  
 $d = 260.906$

Increase the depth of footing by 2 times  $d = 2 \times 260.906 = 521.812 \approx 530$  mm  
provide 60mm effective cover, overall depth ( $D$ ) =  $530 + 60$   
 $= 590 \text{ mm}$

Calculation of area of reinforcement

along Y-Y direction

$$M_{y-y} = 0.87 f_y A_{st} (d - \frac{f_y A_{st}}{f_{ck} b})$$

$$\text{or } 751.3137 \times 10^6 = 0.87 \times 415 \times A_{st} \left( 590 - \frac{415 \times A_{st}}{20 \times 4000} \right)$$

on solving we get.

$$A_{st} = \frac{3643204}{4089.98} \text{ mm}^2$$

$$\text{provide } 16\text{mm } \phi \text{ bar at spacing} = \frac{\pi \times 16^2}{4} \times 4000$$

$$= \frac{3643204}{4089.98} = 196.64 \text{ mm}$$

$$\approx 200 \text{ mm (reduce spacing)}$$

provide 16mm  $\phi$  bar 200mm clc.

$$\text{actual } (A_{st}) \text{ provided} = \frac{\pi \times 16^2}{4} \times 4000$$

$$= \frac{402124}{4089.98} \text{ mm}^2$$

reinforcement along X-X direction

$$M_{x-x} = 0.87 f_y A_{st} x \left( d - \frac{f_y A_{st} x}{f_{ck} L} \right)$$

$$\text{or } 751.3137 \times 10^6 = 0.87 \times 415 \times A_{st} x \left( 590 - \frac{415 \times A_{st} x}{20 \times 4000} \right)$$

on solving

$$A_{st} x = 4089.98 \text{ mm}^2$$

$$\text{provide } 16\text{mm } \phi \text{ bar at spacing} = \frac{\pi \times 16^2}{4} \times 4000$$

$$= 196.64 \text{ mm}$$

$$= 180 \text{ mm}$$

provide 16mm  $\phi$  bar 180mm clc.

check for one way slab.

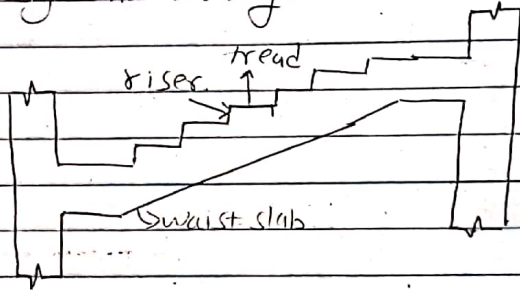
$$V_{y-y} = w \cdot B \left( \frac{L}{2} - \frac{l}{2} - d \right)$$

$$= 108.75 \times 4 \times \left( \frac{4}{2} - \frac{0.282842}{2} - \right)$$

2021 January 16 Sunday

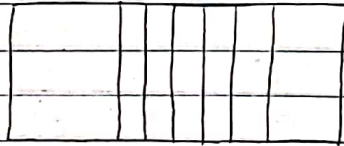
Chapter:- five Design and detailing of miscellaneous structures

Staircase



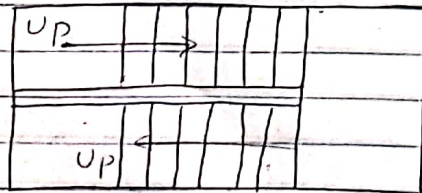
Types of staircase

1) Single flight staircase

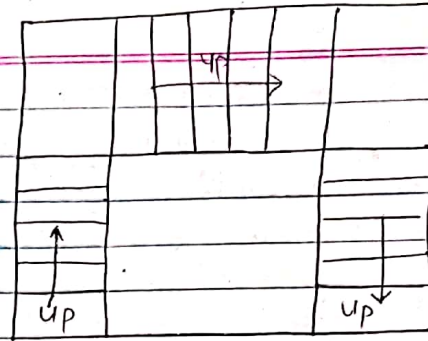


2) Double flight staircase

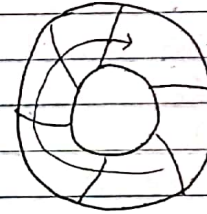
a) Dog legged/well staircase



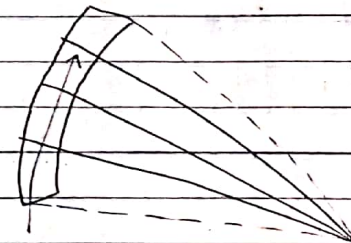
b) Open legged/well staircase



3) Spiral staircase



4) Helicoidal staircase



- staircase spanning longitudinally
- staircase spanning transversely.

- for residential/office building

Riser  $R \rightarrow (150-200)$  mm

Tread  $T \rightarrow (250-300)$  mm



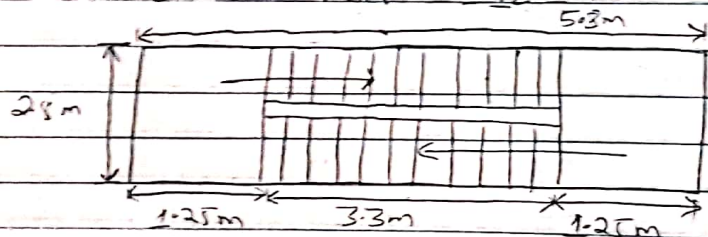
397 Design a dogged legged staircase in a room  $2.8m \times 5.8m$  clear size for an office building assuming floor to floor height of  $3.6m$ , flight width  $1.2m$  and landing width  $1.25m$

Assume the stairs to be supported on  $230mm$  thick masonry wall at the edges of landing parallel to the riser. Use M20 concrete & Fe415 steel assume live load of  $5kN/m^2$ .

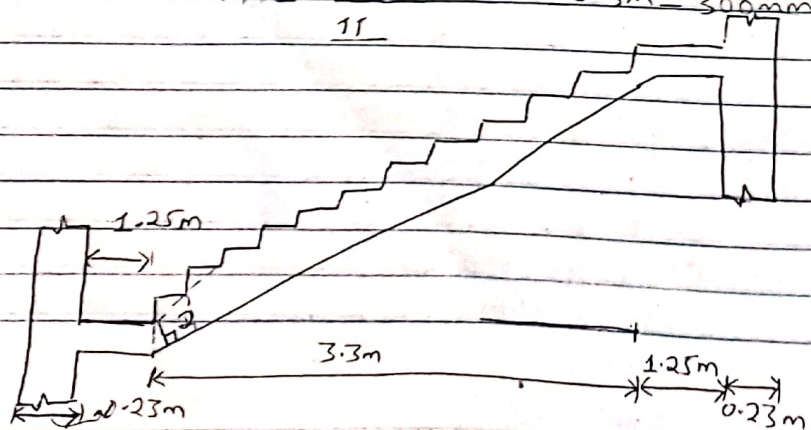
Sol<sup>n</sup>

assuming two flights, height of each flight =  $3.6 - 1.5m$   
 no. of riser =  $\frac{1800}{150} = 12$ . (do not include in clear)

no. of tread = no. of riser - 1 =  $12 - 1 = 11$



width of tread (T) =  $\frac{5.8 - 2 \times 1.25}{11} = 0.3m = 300mm$



$$\text{effective length} = 3.3 + 2 \times 1.25 + 0.23 = 6.03m$$

$$\text{waist slab thickness} = \frac{\text{effective length}}{20 \times \text{modification}} = \frac{6.03}{20 \times 15}$$

Simply supported case

$$\text{providing } 12mm \text{ } \phi \text{ bar at } 15mm \text{ clear cover, overall depth} = \frac{20 + 15 + 12}{2} = 222mm$$

$$\text{Effective depth (d)} = 230 - \frac{12}{2} = 209mm$$

overall depth in landing is reduced to 190mm size B.M in landing is low.

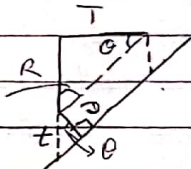
Load calculation -  
 on steps

$$\gamma = 25kN/m^3$$

$$\text{Self wt. of waist slab} = 25 \cdot D \times \frac{\sqrt{R^2 + T^2}}{T} kN/m^2$$

$$= 25 \times 0.23 \times \frac{\sqrt{0.15^2 + 0.3^2}}{0.3} \times 90^\circ - \theta$$

$$= 6.428 kN/m^2 \quad \cos \theta = \frac{D}{T}$$



$$\text{Self wt of steps} = 25 \times \frac{1 \cdot D}{2} \quad t = 2 = D$$

$$= 25 \times \frac{1}{2} \times 0.23 \quad (\text{riser only}) \quad \cos \theta = \frac{T}{\sqrt{R^2 + T^2}}$$

$$= 2.875 kN/m^2 \quad \text{also } \cos \theta = \frac{T}{\sqrt{R^2 + T^2}} = \frac{D}{\sqrt{R^2 + T^2}}$$

$$\text{floor finish (40mm)} = \frac{40}{1000} \times 24 = 0.96 \text{ kN/m}^2$$

$$\text{Live load} = 5 \text{ kN/m}^2$$

$$\text{Total load} = 14.263 \text{ kN/m}^2$$

$$\text{factored load} = 21.394 \text{ kN/m}^2$$

on landing

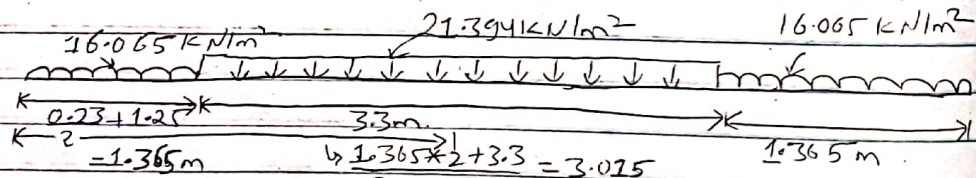
$$\text{Self wt. of waist slab} = 25 \times 0.19 = 4.75 \text{ kN/m}^2$$

$$\text{floor finish} = \frac{40}{1000} \times 24 = 0.96 \text{ kN/m}^2$$

$$\text{Live load} = 5 \text{ kN/m}^2$$

$$\text{Total load} = 10.71 \text{ kN/m}^2$$

$$\text{factored load} = 1.5 \times 10.71 = 16.065 \text{ kN/m}^2$$



$$\text{Reaction (R)} = \frac{16.065 \times 1.365 + 21.394}{2} = 57.228 \text{ kN/m}$$

assuming p.p.m  
i.e. 57.228 kN

$$\text{Max}^m \text{ B.M, } M_{\max} = \frac{57.228 \times 3.075}{2} - \frac{16.065 \times 1.365 \times (3.075 - 1.365)}{2}$$

$$- \frac{21.394 \times 3.3 \times 3.3}{4} = 92.271 \text{ kNm}$$

check for depth,

$$M_{\max} = 0.36 f_{ck} \times \frac{1}{4} b (d - 0.42 x_u)$$

$$\text{or, } 92.271 \times 10^6 = \frac{0.36 \times 20 \times 0.481 \times 1000 \times (d - 0.42 \times 0.481 d)}{4}$$

$\beta = 1m$

on solving we get,

$$d = 187.867 \text{ mm} < 209 \text{ mm}$$

OK

calculation of area of reinforcement

$$M_{\max} = 0.87 f_y A_{st} (d - \frac{f_y A_{st}}{f_{ck} b})$$

$$\text{or, } 92.271 \times 10^6 = 0.87 \times 415 \times x \left( \frac{209 - 415 \times A_{st}}{20 \times 1000} \right)$$

on solving we get

$$A_{st} = 1424.155 \text{ mm}^2$$

$$\text{provide } 10 \text{ mm } \phi \text{ bar at spacing} = \frac{\pi \times 10^2}{4} \times \frac{1000 \text{ mm}}{1424.155}$$

$$= 141.179 \approx 140 \text{ mm}$$

distribution bar - 0.12% of  $b \times d$

$$= \frac{0.12 \times 1000 \times 230}{100} = 276 \text{ mm}^2$$

$$\text{provide } 10 \text{ mm } \phi \text{ bar at spacing} = \frac{\pi \times 10^2}{4} \times \frac{1000}{276} = 284.56 \text{ mm}$$

$$\frac{4 \times 276}{2} \approx 280 \text{ mm}$$

check for shear

$$\text{Ultimate shear stress } (T_u) = \frac{V_u}{b d} = \frac{57.228 \times 10^3}{1000 \times 190} = 0.3012 \text{ N/mm}^2$$

(at support)  $\downarrow$  landing width

$$\% \text{ of steel} = \frac{A_{st}}{b d} \times 100 = \frac{1436.156}{1000 \times 190} \times 100 = 0.755 \%$$

From table  $E_c = 0.56 \text{ N/mm}^2$

$$T_c' = K \cdot T_c \text{ where } K = 1.15$$

$$T_c' = 1.15 \times 0.56 = 0.644 \text{ N/mm}^2 > 0.301 \text{ N/mm}^2$$

OK

Check for development length

$$l_d = \frac{0.87 \cdot f_y \cdot \phi}{4 \cdot T_c'}$$

$$= \frac{0.87 \times 415 \cdot \phi}{4 \times 1.2 \times 1.6} = 47 \phi$$

$$\begin{aligned} \text{Moment of reinforcement (M)} &= 0.87 \cdot f_y \cdot \frac{A_{st}}{2} \left( d - \frac{f_y A_{st}}{2 \cdot k \cdot b} \right) \\ &= 0.87 \times 415 \times \frac{26.156}{2} \left( 209 - \frac{415 \times 26.156}{2 \times 20 \times 100} \right) \\ &= 50.323 \text{ kNm} \end{aligned}$$

Now,

$$l_d \leq 1.3 \frac{M}{V} + l_o$$

$$\text{or } 47 \phi \leq 1.3 \times \frac{50.323 \times 10^3}{57.228}$$

$$\text{or } 47 \phi \leq \frac{1143.145}{47} = 24.322$$

$$\therefore \phi \leq 24.322 \text{ mm}$$

Since actual dia provided is 16mm it is safe in development length

