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CHAPTER :5 LEVEL AND STRUCTURE OF INTEREST RATE

Structure of Interest rate:-

Term structure of interest rate:-

⇒ " " " " " shows the relationship ~~with~~ betⁿ interest rate and maturity period.

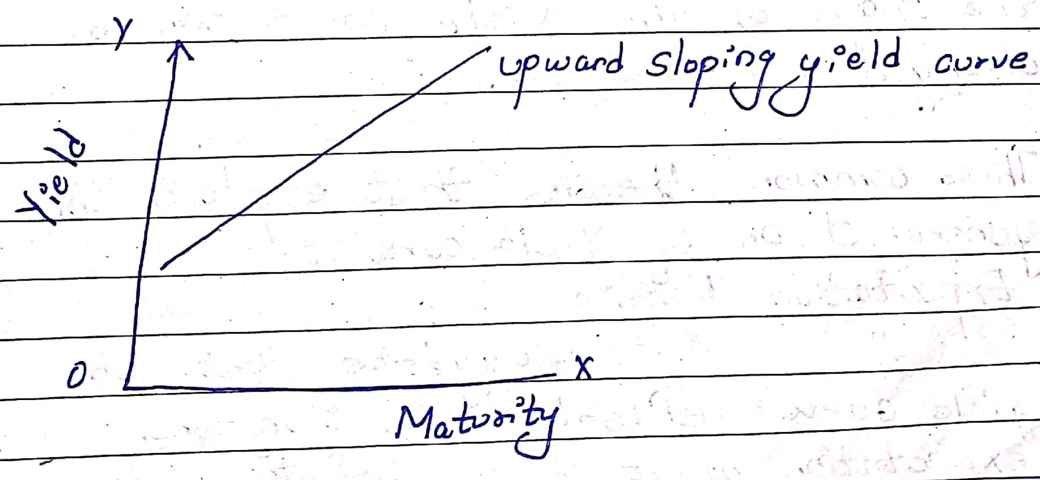
⇒ The relationship between interest rate and maturity period for a given period of time can be presented graphically by a yield curve.

⇒ Generally, there are three types of yield curve:-

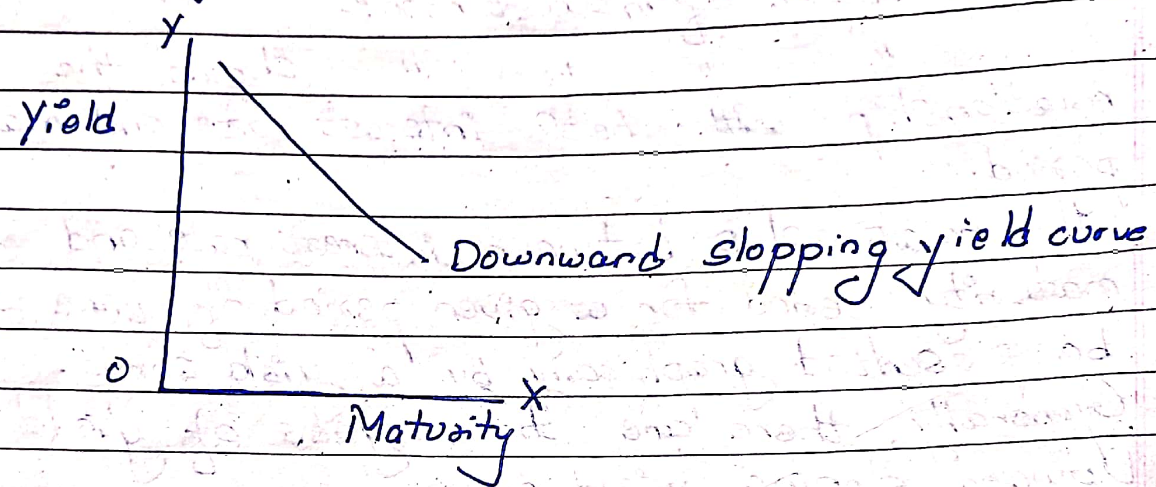
1) Upward sloping yield curve
2) Downward slopping yield curve
3) Horizontal / Flat yield curve

* Upward sloping yield curve:-

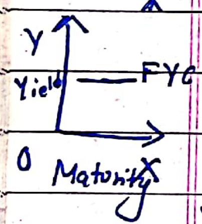
" " " " represents long term securities provide higher yield (return) than short term securities.



* **Downward sloping yield curve:-**
" " " " represent the short term securities provide higher yield than long term securities.



* **Horizontal yield curve:**



" " " represent both short term and long term securities provide same yield.

Determinants of the shape of the term structure or yield curve:
The shape of the yield curve can be changed over the time.

Three common theories that explain the general shape of yield curve:-

1. Expectation theory:
" " suggests that the yield curve reflects the investors expectation about future inflation.

High liquidity → Low interest rate & vv

Zero coupon bond = ~~Dr~~ Pure discount bond

According to ~~this theory~~, increasing inflation expectation, result in an upward sloping yield curve, a decreasing inflation expectation result in a downward sloping yield curve and a stable inflation expectation result in a flat yield curve.

2. Liquidity preference theory :-

Li " " " state that long term securities provide higher yield than short term securities because of liquidity risk premium.

3. Market segmentation theory :-

This theory states that interest rate is determine by demand and supply ~~theory~~ of lonable fund.

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Spot rate and forward rate

Spot rate :-

- Spot rate is the yield to maturity on a pure discount security or zero coupon bond.
- It is the interest rate specified in a spot contract.
- Spot rate represent bond yield from zero period to yields maturity.

Calculation :

Spot rate for n year bond $(0K_n) = \left(\frac{M}{V_0}\right)^{1/n} - 1$

where,

M = Face value of bond n = maturity in year

V_0 = Current price of bond

For example:

Maturity	Face value	Price of bond
1	1000	950
2	1000	930
3	1000	900
4	1000	850

Calculate 1, 2, 3 and 4 year spot rate

Solution:

For 1 year bond,

Face value (M) = Rs 1000

Current price (V_0) = Rs 950

Maturity (n) = 1 yr

Now,

Spot rate for 1 year bond $(OK_1) = \left(\frac{M}{V_0}\right)^{1/n} - 1$

$$= \left(\frac{1000}{950}\right)^{1/1} - 1$$

$$= 5.26\%$$

For 2 year bond,

$$OK_2 = \left(\frac{1000}{930}\right)^{1/2} - 1$$

$$= 3.69\%$$

For 3 year bond,

$$OK_3 = \left(\frac{1000}{900}\right)^{1/3} - 1$$

$$= 3.57\%$$

For 4 year bond,

$$0K_4 = \left(\frac{1000}{850}\right)^{1/4} - 1$$

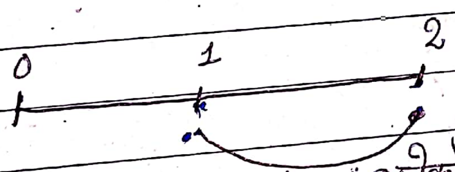
$$= 4.14\%$$

Imp_ Forward Rate: (Future को Rate निकाल्ने)

Forward rate is the interest rate for yield of the bond that is determined today but will be paid money to be borrowed amount at specific date in the future. Forward rate is an interest rate applicable to the financial transaction that will take place in future.

Conditions:-

1. Forward rate for one year bond expected / during 2 year OR forward rate for 1 year bond starting 1 year from now



Expected 2 year अर्नेको अघर 2 year

1 year bond अर्नेको
अघर 1 year back आको

So,
$_1f_2$ निकाल्ने

$$\therefore {}_1f_2 = \left[\frac{(1+0K_2)^2}{(1+0K_1)^1}\right]^{\frac{1}{2-1}} - 1$$

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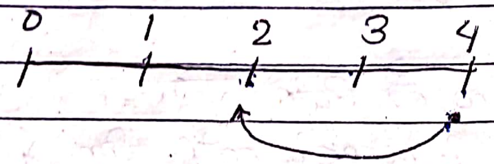
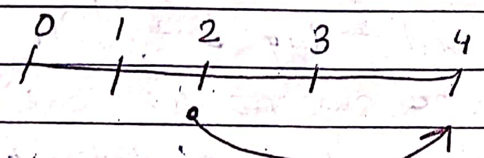
From now अगोत अगे one year back अउले

2 Calculate 2 year ~~forwarded~~ forward rate expected in year 4 or Calculate 2 year forward rate starting 2 year from now.



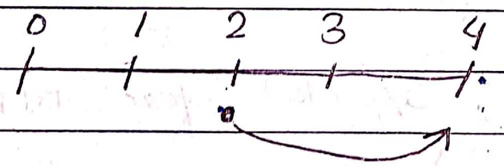
$\rightarrow$ 1st language



$\rightarrow$ 2nd language

$$2f_4 = \left[\frac{(1+0K_4)^4}{(1+0K_2)^2}\right]^{\frac{1}{4-2}} - 1$$

3. Calculate 2 year forward rate starting in 3 year



$2f_4 = ?$

$$2f_4 = \left[\frac{(1+0k_4)^4}{(1+0k_2)^2}\right]^{\frac{1}{4-2}} - 1$$

Calculate 3 year forward rate expected in year 6

$$3f_6 = \left[\frac{(1+{}_0k_6)^6}{(1+{}_0k_3)^3}\right]^{\frac{1}{(6-3)}} - 1$$

Calculate 4 year forward rate starting one year from now.

$$_1k_5 = \left[\frac{(1+{}_0k_5)^5}{(1+{}_0k_1)^1}\right]^{\frac{1}{(5-1)}} - 1$$

Calculate forward rate for 10 year bond starting in year 11

0 1 2 3 4 5 6 7 8 9 10 11 12 20

$$10f_{20} = \left[\frac{(1+{}_0k_{20})^{20}}{(1+{}_0k_{10})^{10}}\right]^{\frac{1}{20-10}} - 1$$

* Current yield on 5 year bond $({}_0k_5)$
  " " " 3 " " $({}_0k_3)$

* Current आयो आनि spot rate

Calculation of spot rate by using forward rate:

Spot rate for 1 year bond $(_0k_1) = {_0F_1}$ (given)

Spot rate for 2 year bond $(_0k_2) = [(1+{_0F_1})(1+{_1F_2})]^{1/2} - 1$

Spot rate for 3 year bond $(_0k_3)$

$= [(1+{_0F_1})(1+{_1F_2})(1+{_2F_3})]^{1/3} - 1$

Spot rate for 4 year bond $(_0k_4)$

$= [(1+{_0F_1})(1+{_1F_2})(1+{_2F_3})(1+{_3F_4})]^{1/4} - 1$

Ques 1: Assume the following zero coupon bond with face value of Rs 1000 that are available

Bond	Current Price	Year to maturity
A	935	1
B	870	2
C	800	3
D	750	4

a) Calculate the spot rate (yield) for each bond

b) " one year forward rate starting in year 2

c) " 2 " " " " " " 3

d) " 3 " " " " " " 2

Solution

a) Spot rate for 1 year bond,

$$_0K_1 = \left(\frac{M}{V_0}\right)^{1/n} - 1$$

$$= \left(\frac{1000}{935}\right)^{1/1} - 1$$

$= 6.95\%$

$$0K_2 = \left(\frac{1000}{870}\right)^{1/2} - 1$$

$$= 7.21\%$$

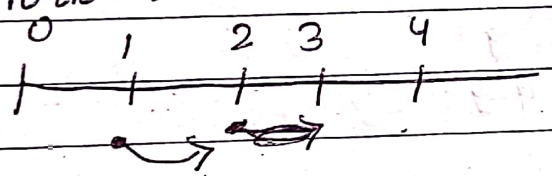
$$0K_3 = \left(\frac{1000}{800}\right)^{1/3} - 1$$

$$= 7.72\%$$

$$0K_4 = \left(\frac{1000}{750}\right)^{1/4} - 1$$

$$= 7.45\%$$

b) Solution:



$$3F_2 = \left[\frac{(1+0K_3)^3}{(1+0K_2)^2}\right]^{\frac{1}{3-2}} - 1$$

$$= \left[\frac{(1+0.0772)^3}{(1+0.0721)^2}\right]^{1} - 1$$

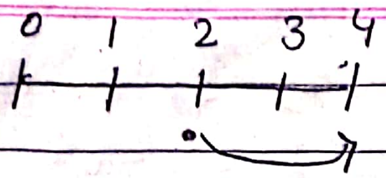
$$=$$

$$1F_2 = \left[\frac{(1+0K_2)^2}{(1+0K_1)^1}\right]^{\frac{1}{2-1}} - 1$$

$$= \left[\frac{(1+7.21)^2}{1+6.95}\right] - 1$$

$$= 7.47\%$$

c)

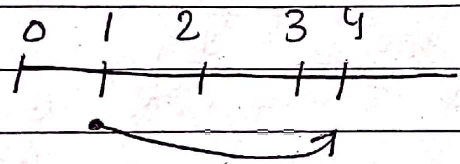


$$2f_4 = \left[\frac{(1+0k_4)^4}{(1+0k_2)^2}\right]^{\frac{1}{4-2}} - 1$$

$$= \left[\frac{(1+7.46)^4}{(1+7.21)^2}\right]^{\frac{1}{2}} - 1$$

$$= 7.71\%$$

d)

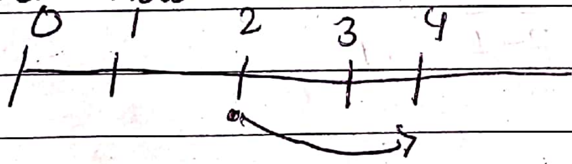


$$1f_4 = \left[\frac{(1+0k_4)^4}{(1+0k_1)^1}\right]^{\frac{1}{4-1}} - 1$$

$$= \left[\frac{(1+7.46)^4}{(1+6.95)^1}\right]^{\frac{1}{3}} - 1$$

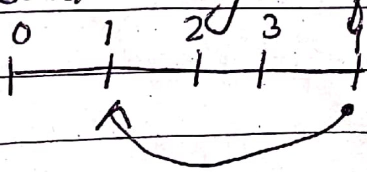
$$= 7.63\%$$

e) Calculate two year forward rate starting 2 year from now



$2f_4$ = निकालो

f) Calculate 3 year forward rate during year 4



$1f_4$ = d मा

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Use the following data :

Bond	Maturity, years	YTM (Spot rate)
W	1	8.0%
X	2	9
Y	3	10.5
Z	4	12.0

a) Calculate the implied 1-year forward rate starting in year 2

0 1 2 3

$$_1f_2 = \left[\frac{(1+0k_2)^2}{(1+0k_1)^1}\right]^{1/2-1} - 1$$

$$= \left[\frac{(1+9)^2}{(1+8)^1}\right]^{1} - 1$$

$$= 10.11\%$$

b) Calculate the implied 1 year forward rate starting in year 3

0 1 2 3 4

$$_2f_3 = \left[\frac{(1+0k_3)^3}{(1+0k_2)^2}\right]^{\frac{1}{3-2}} - 1$$

$$= \left[\frac{(1+10.5)^3}{(1+9)^2}\right]^{1} - 1$$

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c) Solution,

$$1f_4 = \left[\frac{(1+0k_4)^4}{(1+0k_1)^1}\right]^{\frac{1}{4-1}} - 1$$

$$= \left[\frac{(1+12)^4}{(1+8)^1}\right]^{1/3} - 1$$

$$= 13.69\ \%$$

d) $$2f_4 = \left[\frac{(1+0k_4)^4}{(1+0k_2)^2}\right]^{\frac{1}{4-2}} - 1$$

$$= \left[\frac{(1+12)^4}{(1+9)^2}\right]^{1/2} - 1$$

$$= 15.9\%$$

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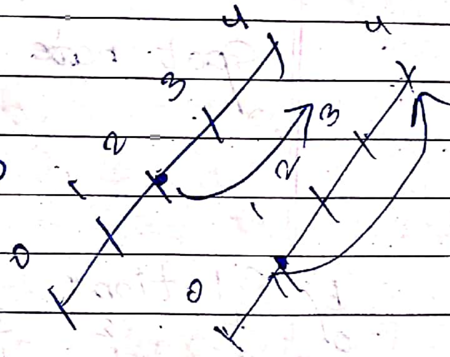
Consider the following information

Bond	Terms	Price
A	1	950
B	2	900
C	3	800
D	4	715



Face value of each bond = Rs 1000

Calculate:

a) Spot rate (YTM) of each bond.

b) Forward rate of 1 year bond during year 3.

c) 2 year forward rate starting in year 2.

d) 3 year forward rate starting in 1 year from now.

a) Solution:

Spot rate of 1 year bond $(OK_1) = \left(\frac{M}{V_0}\right)^{1/n} - 1$

$= \left(\frac{1000}{950}\right)^{1/1} - 1$

$= 5.26\%$

Spot rate of 2 year bond $(OK_2) = \left(\frac{M}{V_0}\right)^{1/n} - 1$

$= \left(\frac{1000}{900}\right)^{1/2} - 1$

$= \cancel{3.57\%}$ 5.40%

Spot rate of 3 year bond (ok_3) $= \left(\frac{M}{V_0}\right)^{1/n} - 1$

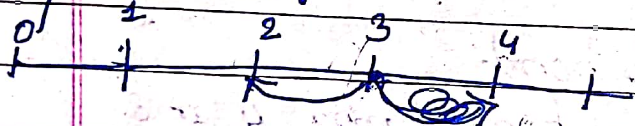
$= \left(\frac{1000}{800}\right)^{1/3} - 1$

$= 7.72\%$

Spot rate of 4 year bond (ok_4) $= \left(\frac{1000}{715}\right)^{1/4} - 1$

$= 8.74\%$

b) Solution:

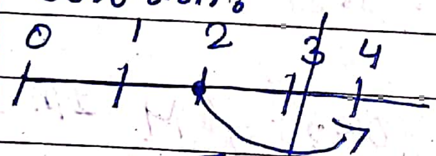


$$_2F_3 = \left[\frac{(1+ok_2)^3}{(1+ok_1)^1}\right]^{\frac{1}{3-1}} - 1$$

$$= \left[\frac{(1+5.4)^3}{(1+5.26)^1}\right]^{1/2} - 1$$

$= \cancel{6.54\%}$ 12.14%

c) Solution:



$$_2F_4 = \left[\frac{(1+ok_4)^4}{(1+ok_2)^2}\right]^{\frac{1}{4-2}}$$

$$= \left[\frac{(1+8.74)^4}{(1+5.4)^2}\right]^{1/2}$$

$= 14.82\%$

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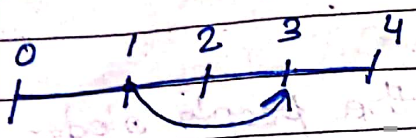
c) Solution:



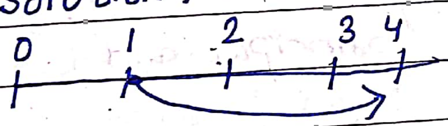
~~$1F_3 = [(1+0$~~

$$2 \text{ year forward rate starting in year } 2 = \left[\frac{(1+0K_3)^3}{(1+0K_1)^1}\right]^{\frac{1}{3-1}} - 1$$

$$= \left[\frac{(1+0.0772)^3}{(1+0.0526)^1}\right]^{\frac{1}{2}} - 1$$

$$= 8.97\%$$

d) Solution:



3 year forward rate starting in year 1 from now

$$= \left[\frac{(1+0K_4)^4}{(1+0K_1)^1}\right]^{1/4-1} - 1$$

$$= \left[\frac{(1+0.0874)^4}{(1+0.0526)}\right]^{\frac{1}{3}} - 1$$

$$= 9.91\%$$

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Mortgage market:

Concept of Mortgage:

A mortgage is defined as the people pledge of physical assets to secure a loan. The physical assets used to secure the loan is generally in the form of real estate such as land and building.

Mortgage market:

A mortgage market is the market that arrange loans for buying property.

Ammortization loan / term loan:

" " refers to the loan that is to be repaired in equal periodic installment including both principal and interest.

a) Monthly payment (MP) $= MB_0 \times \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$

where,

i = monthly interest rate

n = No. of month

MB_0 = original mortgage balance / loan amount

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b) Remaining mortgage balance after t month

$$MB_t = MB_0 \times \left[\frac{(1+i)^n - (1+i)^t}{(1+i)^n - 1}\right]$$

c) Schedule principal at t month $(SP_t) = MB_0 \times \left[\frac{i(1+i)^{t-1}}{(1+i)^n - 1}\right]$

* Suppose a mortgage loan has orginal balance of Rs 3,75,000. The annual interest rate is 8.4 % and mortgage loan has 10 year term.
a) What is the monthly payment on mortgage loan?
b) What is the amount of remaining mortgage balance at the end of 50th month?
c) What is the amount of schedule principal repayment for 60 month and interest amount of this period?
d) Set up amortization schedule for 5 months.

Solution:
Original mortgage balance (MB_0) = Rs 3,75,000
Monthly interest rate $(i) = \frac{0.084}{12} = 0.007$

No. of month $(n) = 10 \times 12 = 120$ month

a) Monthly payment $(M_p) = MB_0 \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$

$$= 375000 \times \left[\frac{0.007(1+0.007)^{120}}{(1+0.007)^{120} - 1}\right]$$

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b) Solution:

Remaining mortgage balance after 50 month

$$MB_{50} = MB_0 \left[\frac{(1+i)^n - (1+i)^t}{(1+i)^n - 1}\right]$$

$$= 375000 \left[\frac{(1+0.007)^{120} - (1+0.007)^{50}}{(1+0.007)^{120} - 1}\right]$$

$= Rs\ 255496.28$

c) Solution:

Schedule principal repayment for 60 month

$$(SP_{60}) = MB_0 \times \left[\frac{i(1+i)^{t-1}}{(1+i)^n - 1}\right]$$

$$= 375000 \times \left[\frac{0.007\ (1+0.007)^{60-1}}{(1+0.007)^{120} - 1}\right]$$

$= Rs\ 3025.03$

Again,

Monthly payment = Principal + Interest

$4629.43 = 3025.03 +$ Interest

$\therefore$ Interest $= Rs\ 1604.4$

d) Loan amortization schedule

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-	a	b	$c = a \times i$	$d = b - c$	$e = a - d$
Month	Beginning Balance	Monthly Payment	Interest	Principal	Ending Balance
1	375000	4629.42	2625	2004.42	372995.58
2	372995.58	4629.42	2610.94	2018.46	370977.12
3	370977.12	4629.42	2596.83	2032.59	368944.53
4	368544.53	4629.42	2582.611	2046.809	366897.64
5	366897.72	4629.42	2568.28	2061.14	364836.5

Consider the following information:

Amount borrowed = Rs 1000,000

Maturity = 240 months

Annual loan rate = 15%

Calculate:

a) What is the monthly mortgage payment?

b) What will be the remaining mortgage balance at the end of 84 ~~240~~ month.

c) What will be the remaining mortgage balance at the end of 240 month.

d) What is the schedule principal for 180 month & interest amount of this period?

e) Prepare loan ammortization shhedule for 4 month.

f) What fraction of payment & principal of 3 months.

g) What fraction of payment & interest of 2 months.

a) Solution:

Original mortgage balance (MB_0) = Rs. 1,000,000

No. of months (n) = 240 months.

Monthly interest rate = $\frac{0.15}{12} = 0.0125$

Monthly payment (MP) = $MB_0 \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$

$$= 1000000 \left[\frac{0.0125(1+0.0125)^{240}}{(1+0.0125)^{240} - 1} \right]$$

$$= Rs\ 13167.89$$

b) Remaining mortgage balance after 84 month

$MB_{84} = MB_0 \left[\frac{(1+i)^n - (1+i)^t}{(1+i)^n - 1} \right]$

$$= 1000000 \left[\frac{(1+0.0125)^{240} - (1+0.0125)^{84}}{(1+0.0125)^{240} - 1} \right]$$

$= Rs\ 901733.13$

c)

d) Solution :

Schedule principal for 180 month (SP_{180})

$$= MBO\left[\frac{i(1+i)^{t-1}}{(1+i)^{n}-1}\right]$$

$$= 1000000\left[\frac{0.0125(1+0.0125)^{180-1}}{(1+0.0125)^{240}-1}\right]$$

= Rs 6171.90

Again,

Monthly payment = Principal + interest

or, 13167.89 = 6171.90 + interest

∴ Interest = Rs 6995.99

e) Loan ammortization schedule

Month	Beginning balance	Monthly payment	Interest	Principal	Ending balance
1	1000000	13167.89	12500	667.89	999332.11
2	999332.11	13167.89	12491.65	676.24	998655.83
3	998655.87	13167.89	12483.19	684.7	997971.17
4	997971.17	13167.89	12474.64	693.25	997277.9

f) Solution:

Fraction payment and principal of 3 months

$$= \frac{\text{Principal 3 month}}{PMT} \times 100\%$$

$$= \frac{684.7}{13167.89} \times 100\%$$

= 5.19

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9) Fraction of payment and interest of 2 month

$$= \frac{\text{Interest of 2nd month}}{PMT} \times 100\%$$

$$= \frac{12491.65}{13167.89} \times 100\%$$

$$= 94.86\%$$

Foreign Exchange:

The market where foreign currency are exchanged