UNIT – 9

SEQUENCE AND SERIES

Definition

A sequence is a function defined on the set of positive integers.

Examples:

i) 3, 6, 9, 12

ii) 2, 4, 8, 16, 32

1) Arithmetic sequence

A sequence is said to be an arithmetic sequence if the difference of two successive terms is always same.

Example: 2, 4, 6, 8, 10, ...

2) Geometric sequence

A sequence is said to be geometric sequence if the ratio of any term to its preceding term is always constant.

Example: 5, 25, 125, 625, ...

3) Harmonic sequence

A sequence is said to be harmonic sequence if the reciprocal of its terms form an arithmetic sequence.

Example: $\frac{1}{4}, \frac{1}{7}, \frac{1}{10}, \frac{1}{13}$,.......

Series

Let a1, a2, a3, ... be a sequence. An expression of the form a1+a2+a3+....... is called a series.

Example: 2+4+6+8+.....

Means

Any terms in between first term and last term of on A.S (or G.S or H.S) called an arithmetic mean (geometric mean or harmonic mean).

Theorem

Given any two numbers a and b, the AM, GM and HM are given by

AM=A=$\frac{a+b}{2}$

GM = G =$\sqrt{ab}$

HM = H = $\frac{2ab}{a+b}$

Proof:

Given a and b be any two numbers. If A be the single AM between a and b, then a, A, b form an AP.

So, A-a=b-A

2A=a+b

$\therefore$ A= $\frac{a+b}{2}$

Again if G be the single GM between a and b, then a, G, b form an GP. So,

$$\frac{G}{a} = \frac{b}{G}$$

or, G^2 = ab

$\therefore$ G =$\sqrt{ab}$

Also, if H be the single HM between a and b, then a, H, b form a HP. So,

$\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ form an AP.

or, $\frac{1}{H}, \frac{1}{a} = \frac{1}{b} - \frac{1}{H}$

or, $\frac{2}{H} = \frac{1}{a} + \frac{1}{b}$

or, $\frac{2}{H} = \frac{a+b}{ab}$

$\therefore$ H = $\frac{2ab}{a+b}$

Theorem

AM, GM and HM between any two unequal positive integers satisfy the following relations:

i) $(GM)^2 = AM \times HM$

ii) $AM > GM > HM$

Proof: Let a and b be two unequal positive numbers, then AM, GM and HM between them are given by:

$AM = \frac{a+b}{2}$

$GM = \sqrt{ab}$

$HM = \frac{2ab}{a+b}$

i) $AM \times HM = \frac{a+b}{2} \times \frac{2ab}{a+b}$

$= ab$

$= (\sqrt{ab})^2$

$= (GM)^2$

$\therefore (GM)^2 = AM \times HM$

ii) $AM - GM = \frac{a+b}{2} - \sqrt{ab}$

$= \frac{a+b-2\sqrt{ab}}{2}$

$= \frac{(\sqrt{a}-\sqrt{b})^2}{2}$

Which is always positive. So,

$AM - GM > 0$

$\therefore AM > GM$ .... (i)

We have,

$GM^2 = AM \times HM$

$\frac{AM}{GM} = \frac{GM}{HM}$

As AM>GM, GM> HM ....(ii)

From (i) and (ii),

we have, AM>GM>HM