Unit -1

1.1. Unit and Measurement

Introduction

The branch of science which deals with correct and precise description of the material universe is known as physics. The word physics comes from a Greek word which means knowledge of nature. To understand physics or any experimental science, we must be able to connect our theoretical description of nature with our experimental observation of nature. Hence for the measurement of the physical quantity we consider a constant quantity as a standard and then find the number which expresses how many times the standard quantity is contained in the physical quantity. This standard is called the unit or the quantity used as a standard measurement is called unit.

Measurement is thus the comparison of an unknown physical quantity with a known fixed unit quantity. For the study unit, we divided it into two categories. They are;

1) Fundamental units
2) Derived units

The units selected for measuring mass, length and time are called fundamental units. These units are independent of each other. The units of physical quantities which can be expressed in terms of fundamental units (mass, length and time) are called derived units, For example; the unit of velocity is a derived unit.

$$\text{Velocity} = \frac{Displacement}{Time} = \frac{Length}{Time}$$

Hence, the unit of velocity can be expressed in the fundamental units of length and time.

Systems of Units

There are several systems of units that are used in different places of World. Some of the common systems of units are as given below:

1. CGS system
2. MKS system
3. FPS system

In the CGS system, the units of length, mass and time are measured in centimeter, gram and second respectively. However, in the MKS system, the units of length, mass and time are meter, kilogram and second respectively. Similarly, for the FPS system, the units of length, mass and time are foot, pound and second respectively.

SI Units

The SI units (System International) were formally introduced in 1960 and have been accepted by all the countries for scientific work. The unit of mass, length and time are not sufficient to describe all the measurements of a system in physics. Hence, seven base quantities of mass, length time, electric current, temperature, luminous intensity and quantity of matter have been assigned proper and standard units known as SI units. In addition to these seven base units, two more supplementary units of plane angle and solid angle have been introduced in this system.

Base Units

Quantity	Symbol	Unit Name	Unit Symbol
Length VV	L	meter	m
Mass	m	kg	kg
Time	t	sec	s
Electric current	I	ampere	A
Temperature	T	Kelvin	K
Luminous Intensity	1	candela	cd
Quantity matter		mole	mol

Supplementary Units

Plane angle	radian	Rad
Solid angle	Steradian	Sr.

Advantage of SI Units

- It is a coherent system of units.
- It is a national system of units
- It is also a decimal system like the CGS and MKS system.
- It is a blend of practical and theoretical work.

Dimensions of a Physical Quantity

Dimensions of a physical quantity may be defined as the powers to which the fundamental quantities in terms of M, L and T for mass length and time respectively must be raised to represent the physical quantity.

For example; the formula for area of a square is (A) = P.

Hence the dimension of the area is [$M^0L^0T^0$]

Examples;

Acceleration $=\frac{Velocity}{Time}$

Dimension of acceleration = $\frac{Dimension\ of\ m/1}{Dimension\ of\ time}$

$$[a] = \frac{= [M^{\circ}LT^{-1}]}{[T]}$$

$$= [M^0L^2T^{-2}]$$

ii. Force = mass × acceleration

$[F] = [M][L T^{-2}]$

$= [MLT^{-2}]$

Dimensional Formulae and Equation

When we express a physical quantity in terms of its dimensions, it is called the dimensional formula of that quantity. For example, the dimensional formula of force is [F] = [MLT^{-2}].

If in an equation containing physical quantities, each quantity is represented by its dimensional formula, the resulting equation is known as dimensional equation.

For example; consider the formula, V = u + at

Writing this formula in terms of dimension, we have;

[M°LT^{-1}] =[M°LT^{-1}] + [M°LT^{-2}] [M°L°T] ....(i)

Equation (i) is known as dimensional equation.

Bridge Course (After SEE)

The dimensional formulae of physical quantities and their SI units.

S. N	Physical Quantity	Relation with other physical quantities	Dimensional formula	SI Unit
1	area	length×breadth	[M°L°T]	m^2
2	volume	L×B×H	[L^3]	m^3
3	density	mass/volume	[$ML^{-3}T^0$]	kg m^{-3}
4	Speed or velocity	distance/time	[$M°LT^{-1}$]	ms^{-1}
5	acceleration	vel./time	[LT^{-2}]	ms^{-2}
6	force	mass×acceleration	[MLT^{-2}]	N
7	moment	mass×velocity	[MLT^{-1}]	kg ms^{-1}
8	work	force×distance	[ML^2T^{-2}]	Nm
9	power	work/time	[ML^2T^{-3}]	W
10	pressure	Force/Area	[$ML^{-1}T^{-2}$]	Nm^{-2}
11	KE	1/2×mass×vel^2	[ML^2T^{-2}]	Nm
12	PE	mass×g×distance	[ML^2T^{-2}]	Nm
13	Impulse	force×time	[MLT^{-1}]	Ns
14	Torque	force×distance	[ML^2T^{-2}]	Nm
15	Stress	Force/Area	[$ML^{-1}T^{-2}$]	Nm^{-2}
16	Strain	Extension in length/original length	[$M°L°T^0$]	Unitless
17	Elasticity	stress/strain	[$ML^{-1}T^{-2}$]	Nm^{-2}

Principle of homogeneity of dimensions

According to this principle, a physical relation is dimensionally correct if each and every term on either side of the equation has the same dimension. For example, consider a physical relation,

$$S = ut + \frac{1}{2}at^2$$

Where,

S is displacement, u is initial velocity, a is uniform acceleration and t is time. So,

Dim. of S = $[M^0LT^0]$

ut = $[M^0LT^\circ]$

at^2 = $[M^\circ LT^\circ]$

Hence, each and every term has the same dimension. Hence the equation is correct.

Uses if dimensional equations

The uses of dimensional equation are given below;

i. To check the correctness of physical relation.

i. To derive relationships between different physical quantities.

iii. To convert one system of units to another.

iv. To find the dimension of constants in a given relation.

Limitations of Dimensional Analysis

- Although dimensional analysis is very helpful in the science of measurements, it has following drawbacks;
- Dimensional constants in a physical formula can not be determined by dimensional analysis.
- Dimensional analysis can not be used to derive relations involving trigonometric and exponential functions.
- Dimensional analysis does not indicate whether a physical quality is a scalar or vector.

1.2. Scalar and Vector

Scalar

The physical quantities having magnitude but not direction are called Scalars. Mass, distance, length, time, density, work, Sp. heat, temp, charge etc are Scalars.

Vectors

The physical quantities having both magnitude as well as direction are called vectors. Displacement, velocity, acceleration, force, momentum, gravitational field, electric field are vectors. Some types of vectors are given below;

i. Equal vectors

Two vectors are said to be equal if they have the same magnitude and the same direction.

ii. Negative vectors

A vector is said to be a negative vector of a given vector if its magnitude is the same as that of the given vector but its direction is opposite.

iii. Unit vectors

A unit vector of a given vector is a dimensionless vector that has a magnitude of 1 and has the same direction as that of the given vector.

iv. Zero vectors

A vector that has zero magnitude is called zero vector or null vector

Addition of Vectors

Vector addition can be done most easily by a graphical method. Suppose we want to add two vectors (let's these are displacement vectors) as shown in fig. (1). To add them graphically, we place the tail of $\vec{Q}$ at the head of $\vec{P}$ on shown in fig. (2). Having moved vector $\vec{Q}$, we draw another vector $\vec{R}$ from that tail of vector $\vec{P}$ to the head of vector $\vec{Q}$. This vector $\vec{R}$ represent the net displacement (or vector sum of $\vec{P}$ and $\vec{Q}$) and is called the resultant of vectors $\vec{P}$ and $\vec{Q}$.

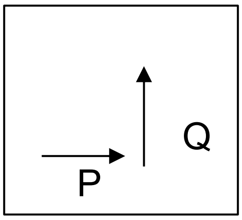


Fig. (1)

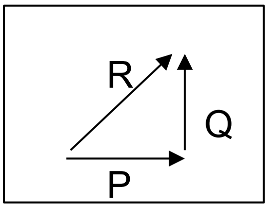


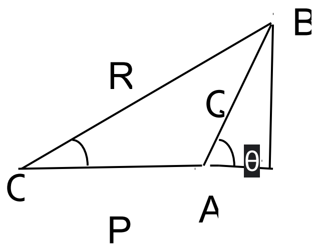
Fig. (2)

Triangle Law of Vector Addition

It states that, "If two vectors acting simultaneously at a point are represented in magnitude and direction by the two sides of a triangle taken in the same order, then the third or closing side of the triangle taken in the opposite order represents their resultant magnitude and direction.

To find magnitude and direction of resultant;

Consider two vectors $\vec{P}$ and $\vec{Q}$ act on a body. Let the vector $\vec{P}$ represents the side A and vector OA and vector $\vec{Q}$ represent the side AB and the angle between $\vec{P}$ and $\vec{Q}$ be θ. Then according to the triangle law of vector addition the resultant vector $\vec{R}$ of $\vec{P}$ and $\vec{Q}$ is the line joining between O and B in figure. Let ϕ be the angle made by the resultant $\vec{R}$ with $\vec{P}$ in figure.



Magnitude of R;

From right angle triangle OCB, OA = P, OB = R, AB = Q

OC = OA + AC

= OA + AB sinθ

$(OB)^2 = (OC) + (CB)^2$

$R^2 = (P+Q \sin\theta) + (Q \cos\theta)^2$

$= p^2 + 2P\,O \sin\theta + Q^2 \sin^2\theta + Q^2\cos^2\theta$

$= P^2 + Q^2 + 2PQ \cos\theta$

$R = \sqrt{(P^2 + Q^2 + 2PQ\cos\theta)}$

Direction of R;

In the triangle OCB,

$\tan\phi = BC / OC = \left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$

$\phi = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$

Parallelogram Law of Vector Addition

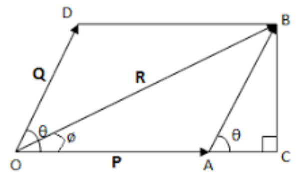
If two vectors acting simultaneously at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then the diagonal of the parallelogram passing through that point represents their resultant magnitude and direction.

For magnitude of resultant;

Let two vectors $\vec{P}$ and $\vec{Q}$ be represented in magnitude and direction by the adjacent sides OA and OB of the parallelogram OACB in figure. Let the angle between $\vec{P}$ and $\vec{Q}$ be θ. Hence, according to parallelogram law of vector addition, the diagonal OC represents the resultant $\vec{R}$ (=0C) in magnitude and direction suppose $\vec{R}$ makes α

with P.

i.e. angle AOC = α



From C, draw CD on OA produced.

In right angle triangle ODC,

$(OC)^2 = (OD)^2 + (CD)^2$

$= \{(OA) + (AD)\}^2 + (CD)^2$

$R^2 = (P+Q\cos\theta)^2 + (Q\sin\theta)^2$

$= P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$

$= P^2 + Q^2(\sin^2\theta + \cos^2\theta) + 2PQ\cos\theta$

$= P^2 + Q^2 + 2PQ\cos\theta$

$$R = \sqrt{(P^2 + Q^2 + 2PQ\cos\theta)}$$

Direction of R;

In the triangle ΛODC,

$\tan\alpha = CD/OD = \frac{CD}{OA+AD} = \frac{Q\sin\theta}{P+Q\cos\theta}$

$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}$

Or, $\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P+Q\cos\theta}\right)$

1.3. Kinematics and Laws of Motion

Kinematics is the branch of mechanics which deals with the study of motion of a body without studying the cause.

Difference between distancé and displacement

Distance	Displacement
- Distance is the length of an actual path traveled by the body. - It should always be positive. - It is a scalar quantity.	- Displacement is the shortest length between initial and final position of the body. - It may be or may not be positive. - It is a vector quantity.

Differences between speed and velocity

Speed	Velocity
- It is distance covered by body per unit time.	- It is the displacement covered by the body per unit time.
- It does not indicate any direction of motion.	- It shows the direction of motion.
- It is a scalar quantity.	- It is a vector quantity.
- It is always positive.	- It may or may not be positive.

Uniform Velocity

An object is said to be moving with a uniform velocity if it undergoes equal displacement in equal intervals of time.

Uniform Acceleration

An object is said to be moving with a uniform acceleration in a plane if its velocity changes by equal amounts in equal intervals of time.

Relative Velocity

Relative velocity is the time rate of change of position of one body with respect to another object. When two bodies A and B are moving in opposite direction with velocity VA and V, then, relative velocity of A with respect to B is,

$$V_{AB} = V_A - (-V_B)$$
$$= V_A + V_B$$

Projectile

A projectile is an object that is thrown into air and then moves under the influence of gravity alone: Some examples of projectile motions are; A bullet fired from the gun, An athlete doing high jump.

Projectile fired at an angle with the horizontal

Let a projectile is projected from the origin O with the initial velocity $\vec{V}_0$ at an angle θ above the x-axis as shown in figure. So, initial velocity in x-axis (horizontal direction) is $v_0\cos\theta$ and in y-axis (vertical direction) is $v_o\sin\theta$.

Let the object is at point P in time t, whose horizontal and vertical distance are x and y. Here, horizontal velocity is not affected by gravity. So, horizontal distance;

$x = v_0\cos\theta \text{ x } t$

$t = x/(v_0\cos\theta)$

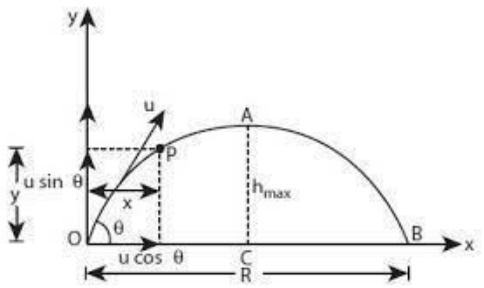
Here, vertical velocity is affected by gravity. So, vertical distance;

$y = v_o\sin\theta \text{ x } t - \frac{1}{2}gt^2$

$= v_o\sin\theta \frac{x}{v0cos\theta} - \frac{1}{2}g\left(\frac{x}{v0cos\theta}\right)^2$

$= x.\tan\theta - \frac{g}{2v02cos2\theta} * x^2$

Above equation is the equation of parabola. Hence, the path of the projectile is parabolic.



Maximum Height (H_{max})

It is the maximum height to which the projectile rises above the launching point. The position of the projectile along axis at any time t is;

$y = (v_o\sin\theta).t - \frac{1}{2}gt^2$

When, $t = \frac{T}{2} = \frac{vosin\theta}{g}$, then $y = H_{max}$

or, $H_{max} = (v_o \sin\theta) \cdot \frac{vosin\theta}{g} - \frac{1}{2}g\left(\frac{vosin\theta}{g}\right)^2$

$H_{max} = v_0^2 \sin^2\theta / 2g$

Horizontal Range (R)

It is the horizontal distance travelled by the projectile before returning to the ground (i.e. original height).

Horizontal range = horizontal velocity x time of flight

$R = (v_o \cos\theta).t$

$R = v_o \cos\theta \times \frac{2vosin\theta}{g}$

$R = v_0^2 . \frac{2sin\theta cos\theta}{g} = v_0^2 . \frac{Sin2\theta}{g}$

Condition for maximum horizontal range

For the given initial speed vo, the horizontal range will be maximum when;

$\sin 2\theta = 1$ or $\theta = 45°$

Max. range $R_{max} = v_0^2/g$

Horizontal Projectile

Suppose a projectile is fired horizontally with velocity vo at a height H above the ground as shown in figure. The horizontal velocity vo remains constant throughout the projectile motion.

The vertical downward velocity is zero at the time of firing the projectile and goes on increasing uniformly with time. Let O be the point of origin from where the projectile is thrown.

Path of projectile

Let the position of the object at P is (×, -y).

Hence, for horizontal distance;

$x = v_o t$ $\quad$ $t = x/v_0$

For vertical distance,

$-y = 0 - \frac{1}{2}gt^2$

$y = \frac{1}{2}gt^2 = \frac{1}{2}g\left(\frac{x}{v0}\right)^2$

$y = \left(\frac{g}{2v0}\right)x^2$

This is the equation of parabola. Hence the path or trajectory of the projectile is parabolic.

Time of flight (T)

If T be the time of flight, then,

$h = O + \frac{1}{2}gt^2$

$T = \sqrt{\frac{2h}{g}}$

Horizontal range (R)

Horizontal range is the distance covered in the horizontal direction of a projectile. Hence,

$R = V_o \times T$

$R = v_0 \times \sqrt{\frac{2h}{g}}$

Velocity of projectile at time t,

At any time t, velocity v of the projectile is,

$V = \sqrt{(v^2 + g^2t^2 - 2v_0 \sin 0°)}$

$V = \sqrt{(v_0^2 + g^2t^2)}$

1.4. Laws of Motion

Inertia

The tendency of a body to maintain its state of rest or uniform motion in a straight line is called inertia of a body. In other words, objects do not change their state of rest or motion unless acted upon by some net external force. The inertia of a body depends upon its mass. The greater the mass of a body, the greater is its inertia. Both mass and inertia are measured in the same units.

Newton's first law of motion

It states that everybody continues in its state of rest or of uniform motion in a straight line unless it is compelled by some external force to change that state. Newton's first law of motion is also called the law of inertia. Inertia of a body is of three types. They are;

i) Inertia of rest

ii) Inertia of motion

iii) Inertia of direction

Examples of law of inertia

In our daily life, there are a large number of examples that illustrate the law of inertia. Few of the are given below;

1. When a moving bus is suddenly stopped, the passengers tend to fall forward: It is because the lower part of the body of the passenger which is in contact with the bus comes to rest but the upper part of the body tends to be in motion due to inertia. As a result, passengers tend to fall forward.
2. When we beat a carpet with a stick, dust particles are removed: It is because the carpet is suddenly set into motion but the dust particles tend to remain at rest due to inertia. Therefore, dust. particles get removed from the carpet.

Linear Momentum

The linear momentum of a body with mass m travelling with velocity $\vec{v}$ is defined as the product of the mass and velocity.

or, linear momentum= $\vec{P}$ = m$\vec{v}$

The direction of linear momentum is the direction of velocity of the body.

The dimensional formula of linear momentum is $[MLT^{-1}]$.

Newton's Second Law of Motion

It states that, "The rate of change of linear momentum of a body with time is directly proportional to the next external force applied on it and this change takes place in the direction of applied force"

or, Applied force α Time rate of change of linear momentum

Let a body of mass m moving with velocity v. Then, linear momentum be that body is $\vec{P} = m\vec{v}$. Let $\vec{v}$ be the external force applied on the body, then according to Newton's second law of motion;

$$\vec{F} \ \alpha \ \frac{\overrightarrow{dp}}{dt}$$

$$F = k \frac{\overrightarrow{dp}}{dt}$$

Where, k is a constant of proportionality and its value is for SI units

So,

$$\vec{F} = \frac{\overrightarrow{dp}}{dt} = m \frac{\overrightarrow{dv}}{dt}$$

$$\vec{k} = m \,.\, \vec{a}$$

The magnitude of the force is F = ma.

Newton's Third Law of Motion

It states that, "to every action there is equal and opposite reaction"

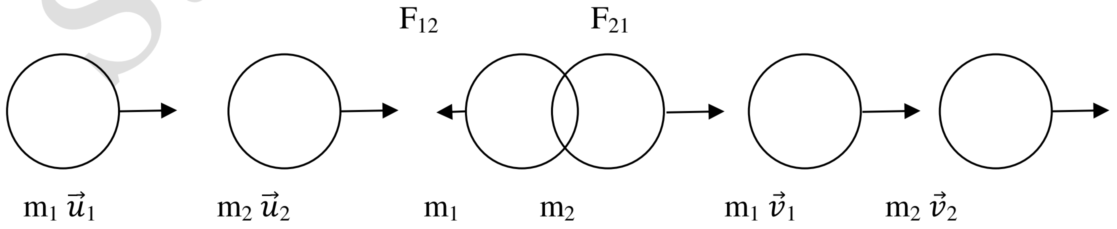
Impulsive Force

The forces which act on bodies for a short time are called impulsive forces. Some examples of impulsive forces are;

- A bat hitting the ball.
- The firing of a gun.

Principles of Conservation of Linear Momentum

According to this law, if the net external force acting on a system of objects is zero, then the vector sum of linear momentum of these objects is constant (conserved), both in magnitude and direction.



Consider two bodies of masses m, and my moving in the same direction with velocity $\vec{u}_1$ and $\vec{u}_2$ respectively as shown in figure. After collision, their velocities become $\vec{v}_1$ and $\vec{v}_2$ respectively.

Hence, the impulse experienced by m, will be $F_{12}.\Lambda t = m_1 \vec{v}_1 - m_1 \vec{u}_1$

Similarly, the impulse experienced by my will be $F_{21}.At = m_2 \vec{v}_2 - m_2 \vec{u}_2$

According to Newton's third law of motion,

$\vec{F}_{12} = - \vec{F}_{21}$

or, $(m_1 \vec{v}_1 - m_1 \vec{u}_1) = - (m_2 \vec{v}_2 - m_2 \vec{u}_2)$

or, $m_1u_1+m_2u_2 = m_1v_1 + m_2v_2$

or, Total linear momentum before collision = Total linear momentum after collision.

Circular Motion

Motion of a body moving in a circle is called circular motion. If the motion of the body is in constant speed, then the circular motion is called uniform circular motion.

Angular displacement and Velocity

The angular displacement of the object moving in a circular path is the angle traced out by the radius vector at the centre of the circular path. The rate of change of angular displacement is called angular velocity and it is denoted by ω.

Centripetal Force

The force that is required to move an object with a constant speed in a circle is called centripetal force.

Here, centripetal force $f_c = m \times a_c$

Where, a_c is the centripetal acceleration

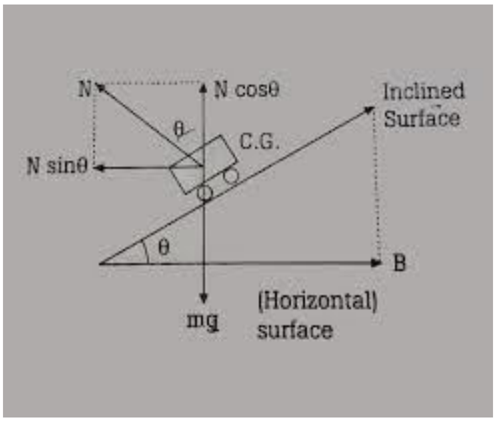
$= m\frac{v^2}{r}$

or, $f_c = mr\omega^2$

$V = \omega r$

Banking of Roads

When an automobile goes around a curve, the road must exert an inward centripetal force on the automobile in order to move it in the circular path. This force is provided by the friction between the tyres and the road. The process of raising the outer edge of a curved road above the level of the inner edge is called banking of the road.



Expression for Banking Angle

Figure shows curved road AB of radius r banking at an angle 0.

Suppose a car goes around this curve with a speed v.

From figure,

$$R \cos\theta = mg \text{............. (i)}$$

The horizontal component R $\sin\theta$ acts inward so that it provides the necessary centripetal force to keep the car in the circular track.

$$R \sin\theta = m\frac{v^2}{r} \text{..........(ii)}$$

Dividing equation (ii) by (i),

$$\tan\theta = m\frac{v^2}{rg}$$

$$\text{or, } \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

1.5. Work, Energy and Power

Work

In physics, work is only done when the force acting on an object produces displacement in it in the direction of force, Suppose a force $\vec{F}$ acts on a body to have displacement d then, work done is;

$$W - \vec{F} \cdot \vec{d}$$

Its unit is Nm or Joule in SI unit.

Kinetic Energy

Energy possessed by a body due to its motion is called kinetic energy. Hence, for more speed, the body has a large KE. Mathematical expression for KE is:

$KE = \frac{1}{2}mv^2$

Potential Energy

Energy possessed by a body due to its position is called potential energy. Mathematical expression for PE is;

PE = mgh

Principle of Conservation of Energy

According to this principle, the total energy of an isolated system is constant. Or the total energy neither increases nor decreases in any process.

Energy Conservation for Freely Falling Body

Let us take an object of mass m at a point A, height h from the ground.

Body is at rest at A. hence,

PE= mgh

KE = 0

So, total energy = PE + KE

= mgh

When the body is falling freely from A, for point B, which is (h -x) height above the ground.

Hence, at B,

PE = mg(h- x)

Velocity at B is;

$V_B = u^2 + 2gx$

$= 2gx$

$KE = \frac{1}{2} m^2gx$

$= mgx$

Hence, total energy at B is,

KE + PE

= mgx + mgh - mgx

= mgh

Similarly, point C,

Velocity $V_c^2 = 2gh$

So. KE = $\frac{1}{2}mV_c^2$ = mgh

and PE = mgh = 0

So, total energy at C = mgh

Thus, total mechanical energy of the body remains the same at all points during the falling process.

Power

The rate at which an agent can do work is called its power.

i.e. power (P) = $\frac{W}{t}$

$P = \vec{F} \cdot \frac{\vec{d}}{t} = \vec{F}.\vec{v}$

Its unit is watt in SI unit and dimensional formula is $[ML^2T^{-3}]$.

Collisions

A collision is a short-time event and is said to have occurred if two (or more) bodies physically collide against each other or even when the path of motion of one particle is affected by the other.

There are two types of collision. viz.

1) Elastic collision

2) Inelastic collision

A collision in which kinetic energy is conserved is called an elastic collision. Elastic collision has the following characteristics;

i. The linear momentum is conserved.
ii. Total energy of the system is conserved.
iii. Kinetic energy is conserved.

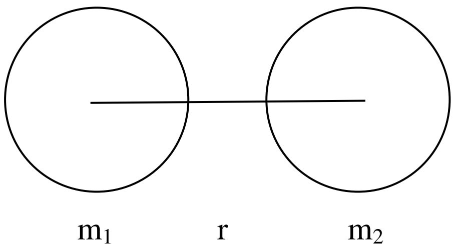
A collision in which kinetic energy is not conserved is called an inelastic collision. Inelastic collision has following characteristics;

i. Linear momentum is conserved.
ii. Total energy of the system is conserved.
iii. Kinetic energy is not conserved.

1.6. Gravity and Gravitation

Newton's universal law of gravitation

Newton's universal law of gravitation states that, "Everybody in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of distance between their centres."



Let us consider two bodies of masses m, and my that have a distance r between their centres as shown in figure. According to Newton's law of gravitation, the magnitude of attractive force (F) between the two bodies is;

$F \, \alpha \, m_1 m_2$..................(i)

$F \, \alpha \, \frac{1}{r^2}$.....................(ii)

Combining equation (i) and (ii),

We have, $F \, \alpha \, \frac{m1m2}{r^2}$

$F = G \frac{m1m2}{r^2}$

Where, G is a constant of proportionality and is called a universal gravitational constant. In SI units, the value of G is;

$G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

Dimension of G

Here, $F = G \frac{m1m2}{r^2}$

$$G = \frac{F.r^2}{m1m2}$$

$$[G] = \frac{[MLT^{-2}][L^2]}{[M^2]} = [M^{-1}L^3T^2]$$

Gravity

Gravity is the force due to earth on a body lying on or near to the earth surface.

Acceleration due to gravity

The acceleration produced on a body due to earth's gravity is called acceleration due to gravity and denoted by g.

Expression of g

From Newton's law of gravitation,

$$F = G\frac{Mm}{R^2}\text{.............(i)}$$

If g is the acceleration produced by the gravitational force, then.

F = gm.................(ii)

or, $g = \frac{F}{m}$

or, $g = \frac{GM}{R^2}$..................(iii)

SI unit, g is measured in ms^{-2}.

Dimension of g

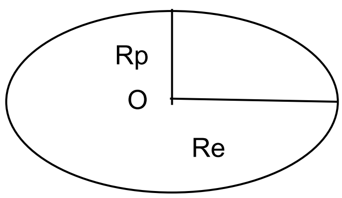
From Newton's law of gravitation,

$$[g] = \frac{[G][M]}{[R^2]} = \frac{[M^{-1}L^3T^2\quad][M]}{[R^2]} = [M^0LT^2]$$

Variation of Acceleration due to Gravity

a) Shape of earth

The earth is not perfectly spherical. It has bulged at the equator and flattered at the pole. So, from the figure, equatorial radius R_e is more than the polar radius R_p. Hence,



$$g = \frac{GM}{R^2}$$

or, $g \,\alpha\, \frac{1}{R^2}$

As, $R_e > R_p$

Therefore, $g_p > g_e$

So; acceleration due to gravity is maximum at the pole and minimum at the equator.

b) Due to height

Let an object of mass m at a point p at the height h from the earth's surface as shown in figure. Hence, acceleration due to gravity, g at the earth's surface is;

$$g = \frac{GM}{R^2} \text{..................(i)}$$

And at point P, acceleration due to gravity g';

$$g' = \frac{GM}{(R+h)^2} \text{................(ii)}$$

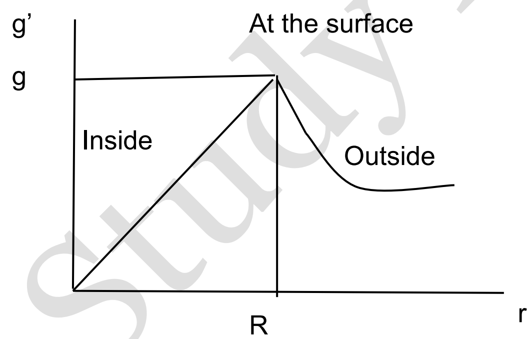
Dividing equation (ii) by (i), we get;

$$\frac{g'}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}} = \frac{R^2}{(R+H)^2}$$

Therefore, $g' = g \cdot \frac{R^2}{(R+h)^2}$.............(iii)

$$g' = \frac{g}{\frac{(R+h)^2}{R^2}}$$

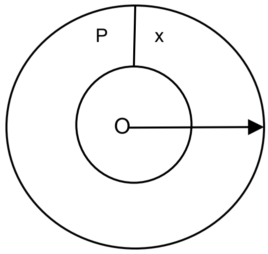
$$= g\left(1 - \frac{2h}{R}\right)$$



Acceleration due to gravity is smaller at the height than at the surface of the earth.

c) Variation of g with depth

Let us consider earth to be a perfect sphere of radius R and uniform density δ. Let P be the point at a distance x from the earth surface.



Mass of earth = M $\frac{4}{3}$= πR^3

Acceleration due to gravity on earth's surface

$$g = \frac{Gm}{Re} = G\,\frac{\frac{4}{3}}{R^2}\pi R^3\delta$$

$$= \frac{4}{3}\pi RG\delta\ldots\ldots\ldots\ldots(ii)$$

Mass of sphere of radius (R-x) be M'

Then, M' = $\frac{4}{3}\pi(R-x)^3\delta\ldots\ldots\ldots(ii)$

$$g' = \frac{Gm}{R^2} = \frac{G4/3\pi(R-x)^3\delta}{(R-x)^2}$$

$$g' = G\frac{4}{3}\pi(R-x)\delta\ldots\ldots\ldots\ldots\ldots(iv)$$

Dividing (iv) with (ii)

$$\frac{g'}{g} = \frac{G4/3\pi(R-x)^3\delta}{G4/3\pi R\delta} = \frac{R-x}{R} = 1- \frac{x}{R}$$

$$g' = g\left(1 - \frac{x}{R}\right)$$

$$g' < g \qquad \left\{\left(1 - \frac{x}{R}\right) < 1\right\}$$

Hence, when we go inside earth, acceleration due to gravity goes on decreasing. At the centre of earth, acceleration due to gravity is zero. If we plot the graph between acceleration due to gravity and the distance out from the centre of earth, the figure as shown in figure obtained.

Gravitational Field

The gravitational field due to a material body is the space around the body in which any other mass experiences a force of attraction. Theoretically, the gravitational field due to a material body extends up
to infinity.

Intensity of Gravitational Field

The intensity of a gravitational field at a point in a gravitational field is defined as the force per unit mass acting on a test mass placed at that point. It is a vector quantity and denoted by the E and R dimensional formula is $[M^0LT^{-2}]$.

Gravitational Potential

The work done in bringing a body of unit mass from infinity to the point considered without acceleration is defined as the gravitational potential at that point. Gravitational potential is a scalar quantity.

1.7 Elasticity

The property of a body by virtue of which it tends to regain its original shape and size when deforming force is removed is called elasticity. A body which regains its exact original shape and size immediately after the removal of the deforming force is called a perfectly elastic body. A body which does not have any tendency at all to regain its original shape and size on the removal of the slightest deforming force is called a perfectly inelastic or plastic body.

Stress

The restoring force per unit area of the body is called stress.

$$\text{Stress} = \frac{External\ force}{Area}$$

Dimensional formula of stress = $[ML^{-1}T^{2}]$

Strain

The change in size or shape of a body due to the deforming force is called strain.

$$\text{Strain} = \frac{Change\ in\ dimension}{Original\ dimension}$$

It is a pure number and has no unit or dimensions.

Hooker's Law

Hooker's law states that, "within elastic limit, the stress developed is directly proportional to the strain produced in a body."

i.e. Stress α Strain

or, Stress = E strain

Therefore, E= Stress/Strain

Where E is a constant of proportionality which is called modulus of elasticity of the material.

Types of Moduli of Elasticity

i) Young's modulus of elasticity

ii) Bulk modulus of elasticity

iii) Shear modulus or modulus of rigidity

Poison's Ratio

When the wire is suspended from one end and loaded at the other end, the length of the wire increases and its diameter decreases. Thus, change in length with original length is called longitudinal strain and change in diameter with original diameter is called lateral strain.

The ratio of lateral strain to the longitudinal strain is called Poisson's ratio (σ)

$$\sigma = \frac{Lateral\ strain}{Longitudinal\ strain} = \frac{-\Delta D/D}{\Delta L/L}$$

$$\sigma = \frac{-\Delta D}{D}\frac{L}{\Delta L}$$

Poisson's ratio is never more than 0.5.

Energy stored in a stretched wire

When the wire is stretched, some external work is to be done against the internal restoring forces. This external work done is the same as elastic potential energy of wire. Consider a wire of length 1 and cross section area A suspended in a rigid supporter in one end and attached mass ion another end as shown in figure. If 1 be the length through which the wire is stretched.

We have,

Longitudinal strain = $\frac{l}{L}$

Stress = $\frac{F}{A}$

Then, Young's modulus of elasticity, Y = $\frac{Stress}{Strain}$

$$\text{or, } Y = \frac{F/A}{l/A} = FL/Al$$

$$\text{or, } F = \frac{Y\,Al}{L}$$

If the wire is stretch through small length dl, then small work done dW is given by,

$$dW = F.dl$$

$$= \frac{Y\,Al}{L}dl$$

Total work done to stretch the wire,

$$W = \int dW$$

$$= \int \frac{Y\,Al}{L}\,dl$$

$$= \frac{Y\,A\,l^2}{L} \cdot \frac{1}{2} = \frac{1}{2}F.l$$

or, $W = \frac{1}{2}\,\frac{F}{A} \cdot \frac{l}{L}A.L$

$$W = \frac{1}{2}\text{ stress. strain. volume}$$

Hence, energy stored in a wire = work done

$$= \frac{1}{2}\text{ stress. strain. Volume}$$