



COMBINATIONAL CIRCUITS



COMBINATIONAL LOGIC



- The output of this type of logic is dependent solely on its current inputs.
- When certain input values are set, a combinatorial circuit generates output values corresponding to those input values. When the input of the combinatorial logic are changed, the outputs are changing to reflect the changes in the new input values.
- Previous values of the inputs do not matter, the current outputs depend solely on the current inputs.
- It's assumed that Circuits are without time delay





1. Design a circuit from a specification.
2. From the specifications of the circuit, determine the required number of inputs and outputs, and assign a letter symbol to each.
3. Derive the truth table that defines the required relationship between inputs and outputs.
4. Obtain the simplified Boolean functions of each outputs as function of the input variables
5. Draw logic diagram and verify correctness



HALF ADDER

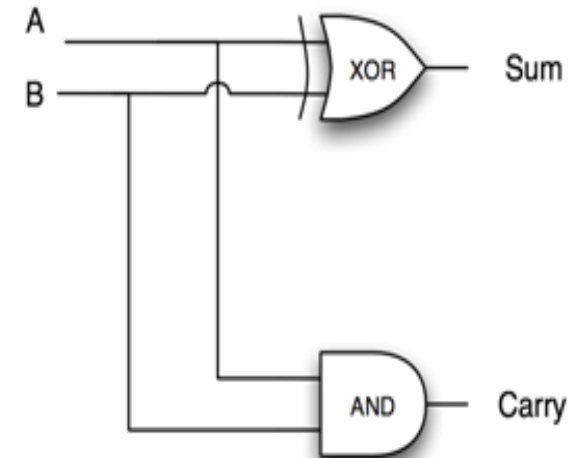
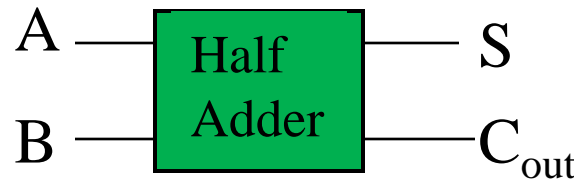


- Logic gate that perform arithmetic addition for 1-bit
- Two inputs A, B to half-adder. Resultants are Sum(S) and Carry(C_{out})
- A wider than 1 bit adder can't use this circuit, because there is no way to input carry information from the previous bits

$$S = \bar{A}B + A\bar{B}$$

$$= A \oplus B$$

$$C = AB$$

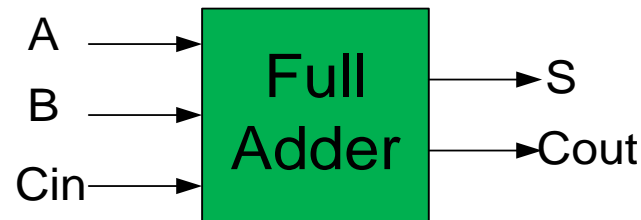


A	B	S	C _{out}
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1





- A full adder is a circuit that computes the sum of three bits and gives a two-bit answer.
- The full adder for a given column adds two bits from the input numbers together with a one-bit carry from the previous column to the right. The adder produces a two-bit answer; one of these bits is used as a carry into the next column.



FULL ADDER



- A full adder has 3 inputs and 2 outputs
- The truth table of the full-adder can be drawn with inputs A,B and Cin with outputs S and Cout.

A	B	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1





- From the truth table we can write the Boolean equation for the S and Cout

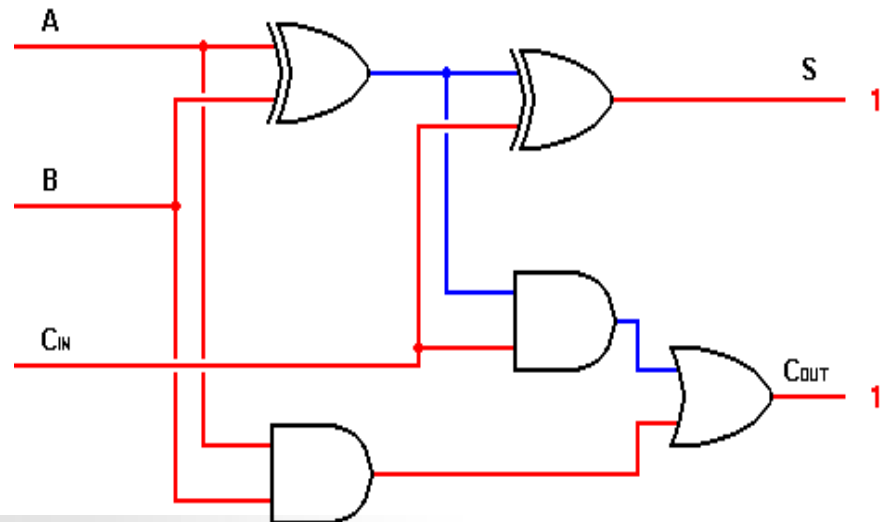
$$S = \bar{A}\bar{B}C_{in} + \bar{A}B\bar{C}_{in} + A\bar{B}\bar{C}_{in} + ABC_{in}$$

$$C_{out} = \bar{A}BC_{in} + A\bar{B}C_{in} + ABC\bar{C}_{in} + ABC_{in}$$

- Simplify using Boolean Algebra and K-map, we get

$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = (A \oplus B)C_{in} + AB$$



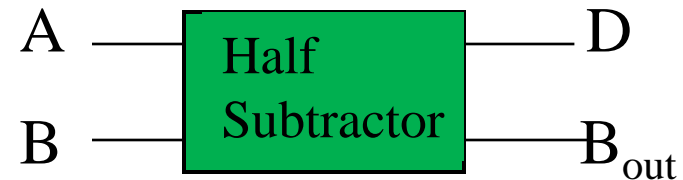
HALF SUBTRACTOR



- Logic gate that perform arithmetic subtraction for 1-bit
- Two inputs A, B to half-subtractor. Resultants are Difference(D) and Borrow(B)

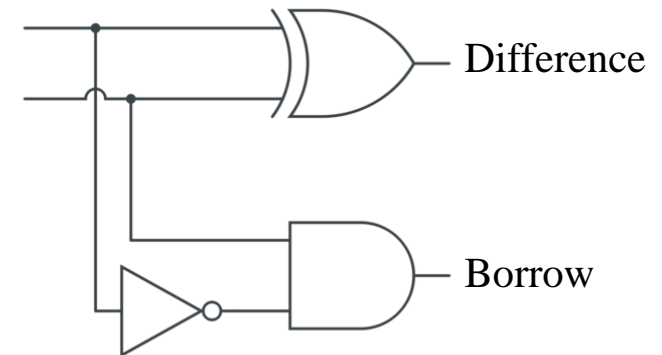
$$D = \bar{A}B + A\bar{B}$$

$$B_{out} = \bar{A}B$$



A	B	D	B _{out}
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

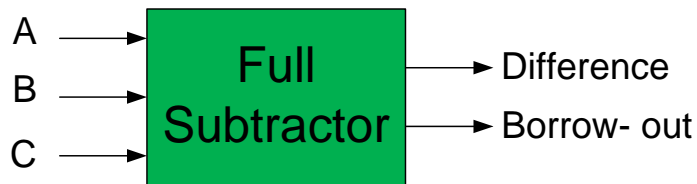
10
-1
1



FULL SUBTRACTOR



- A full subtractor is a circuit that computes the difference of three bits and gives a two-bit answer.
- A full subtractor has 3 inputs and 2 outputs
- The truth table of the full- subtractor can be drawn with inputs A,B and C with outputs D and Bout.



A	B	C	D	Bout
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1



FULL SUBTRACTOR



- From the truth table we can write the Boolean equation for the D and Bout

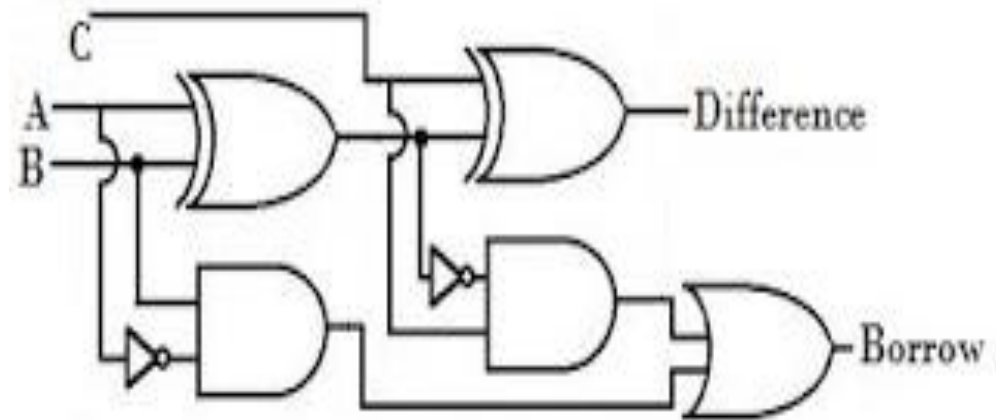
$$D = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

$$B_{out} = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + ABC$$

- Simplify using Boolean Algebra and K-map, we get

$$D = A \oplus B \oplus C$$

$$B_{out} = \overline{(A \oplus B)}C + \bar{A}B$$



COMBINED HALF ADDER/SUBTRACTOR



- **X** = Control signal (not involved in arithmetic operation)
 - 0-add 1-subtract
- **A** and **B** inputs
- **S/D** and **Cout/Bout** outputs

	X	A	B	S/D	Cout/Bout
X=0 ADD	0	0	0	0	0
	0	0	1	1	0
	0	1	0	1	0
	0	1	1	0	1
X=1 SUBTRACT	1	0	0	0	0
	1	0	1	1	1
	1	1	0	1	0
	1	1	1	0	0



COMBINED HALF ADDER/SUBTRACTOR



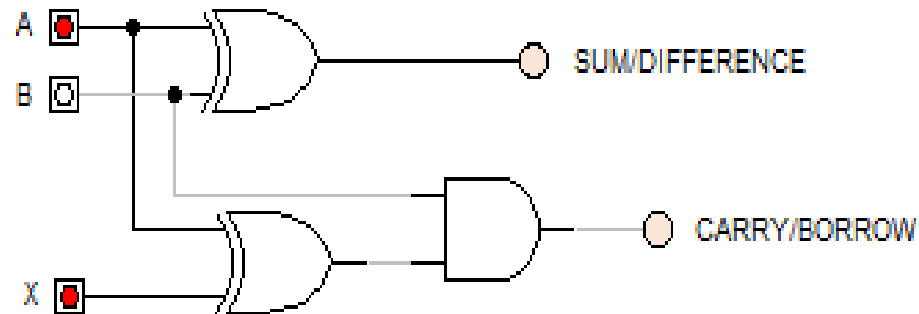
$$S / D = \overline{X} \overline{A} B + \overline{X} A \overline{B} + X \overline{A} B + X A \overline{B}$$

$$C_{out} / B_{out} = \overline{X} A B + X \overline{A} B$$

Simplifying,

$$S / D = A \oplus B$$

$$C_{out} / B_{out} = (A \oplus X) B$$

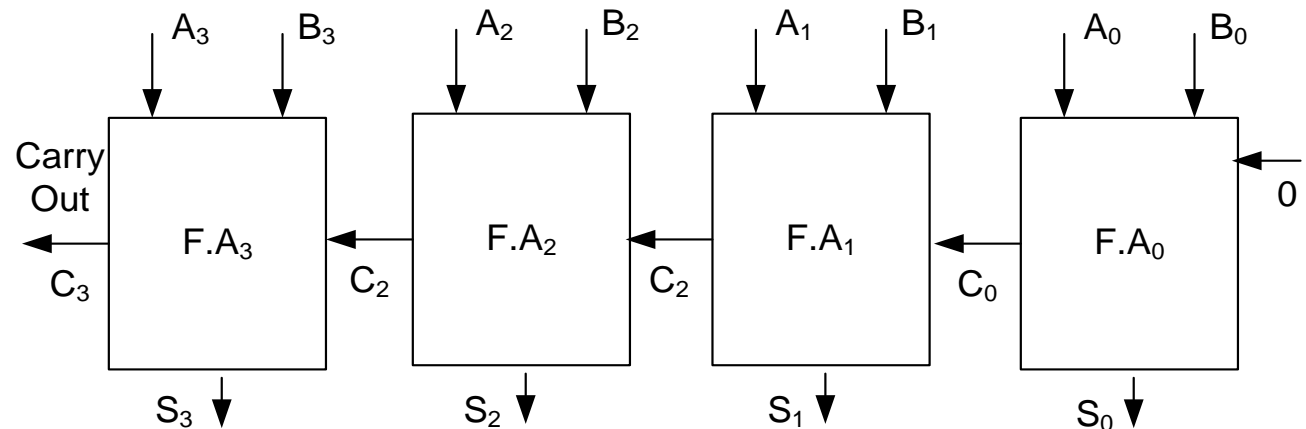


PARALLEL ADDER-4 BIT ADDER



- This circuit is sometimes referred to as a ripple-through adder
- C_0 ripples through four two-level logic circuits and hence the sum cannot be completed until eight gate delays
- For this kind of adder, the maximum delay is directly proportional to the number of stages n .

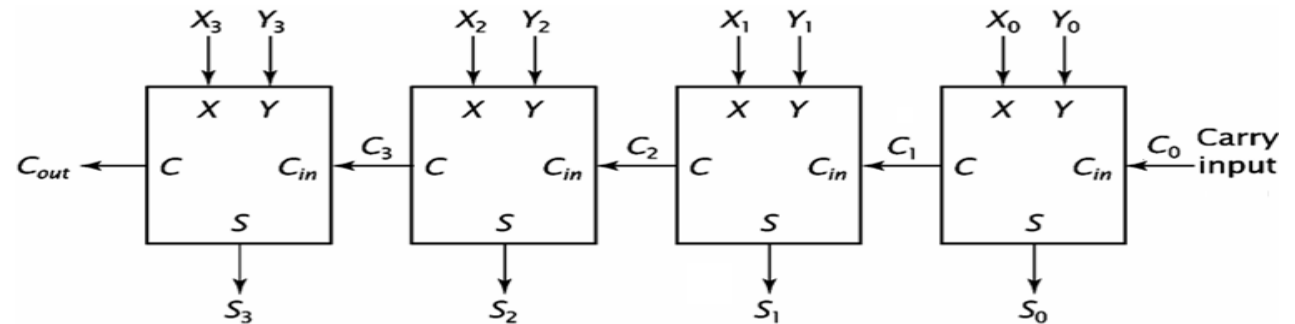
C	1	1	1	0
A	0	1	0	1
B	0	1	1	1
S	1	1	0	0



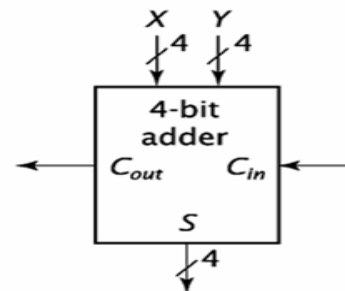
N-BIT ADDER



- With the carry input, full adders can be cascaded to produce an n bit adder by connecting output C from one adder to input C_{in} of the next adder
- Such an adder is called Ripple adder (because the bits ripple through the adder).



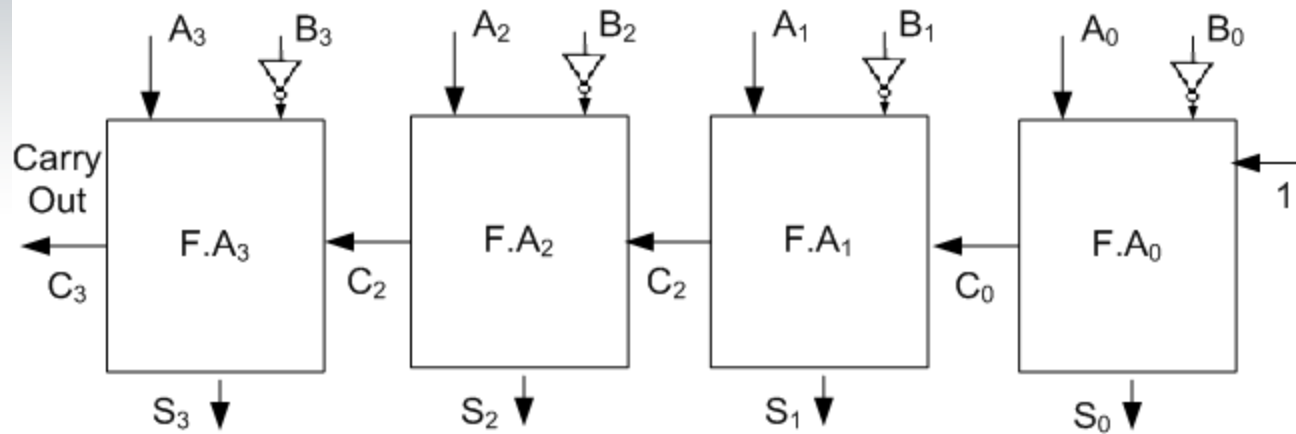
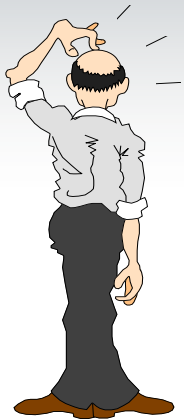
(a)



(b)



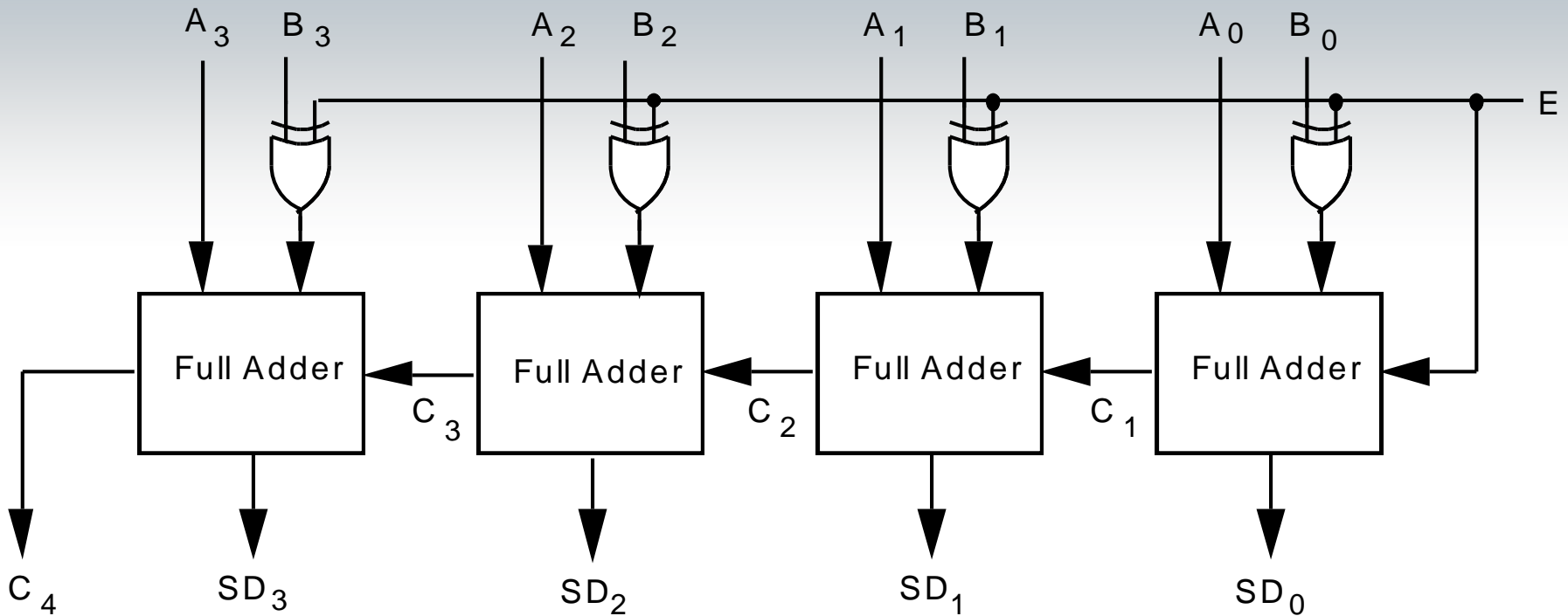
N-BIT ADDER AS A PARALLEL SUBTRACTOR



C_2	C_1	C_0	
A_3	A_2	A_1	A_0
$\overline{B_3}$	$\overline{B_2}$	$\overline{B_1}$	$\overline{B_0}$
		+	1
C_3	S_3	S_2	S_1
			S_0



COMBINED N-BIT ADDER/SUBTRACTOR



$E = 0$: 4-bit Adder

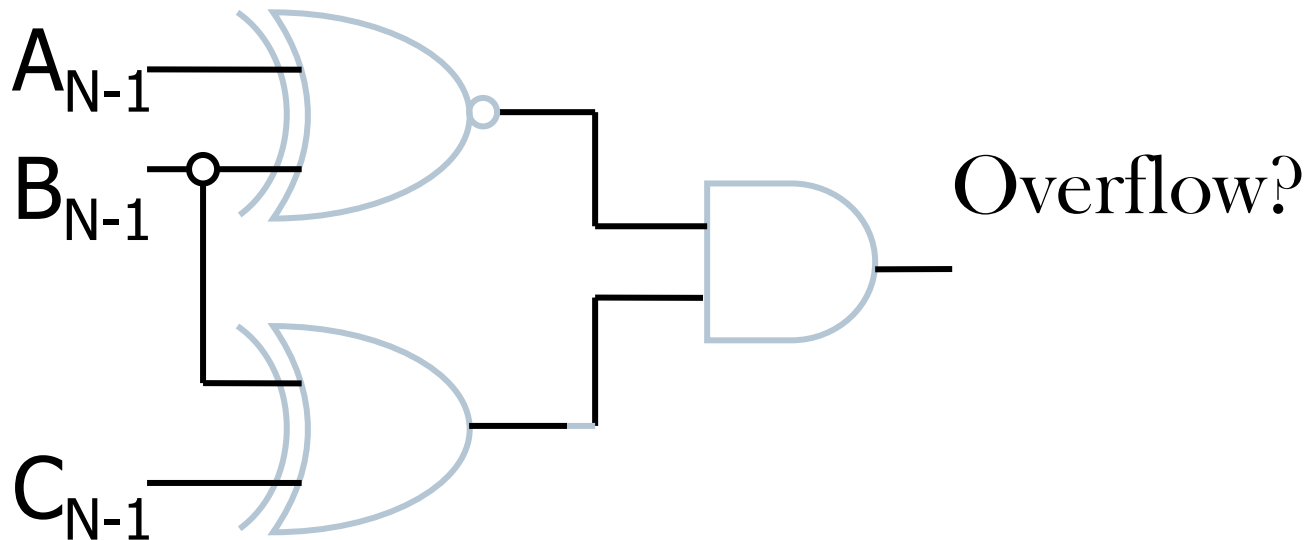
$E = 1$: 4-bit Subtractor



OVERFLOW IN TWO'S COMPLEMENT ADDITION



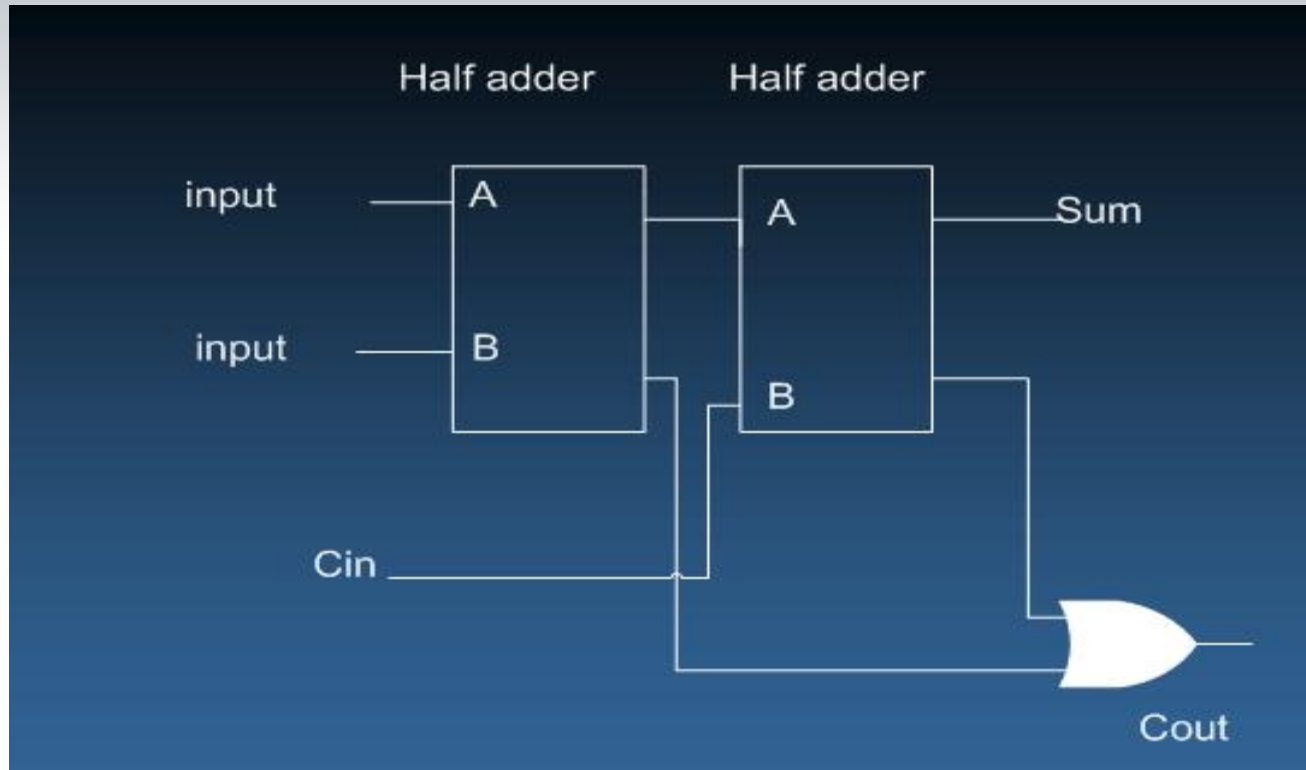
- When two values of the same signs are added:
 - Result won't fit in the number of bits provided
 - Result has the opposite sign.



Assumes an N-bit adder, with bit N-1 the MSB



FULL ADDER USING HALF ADDERS

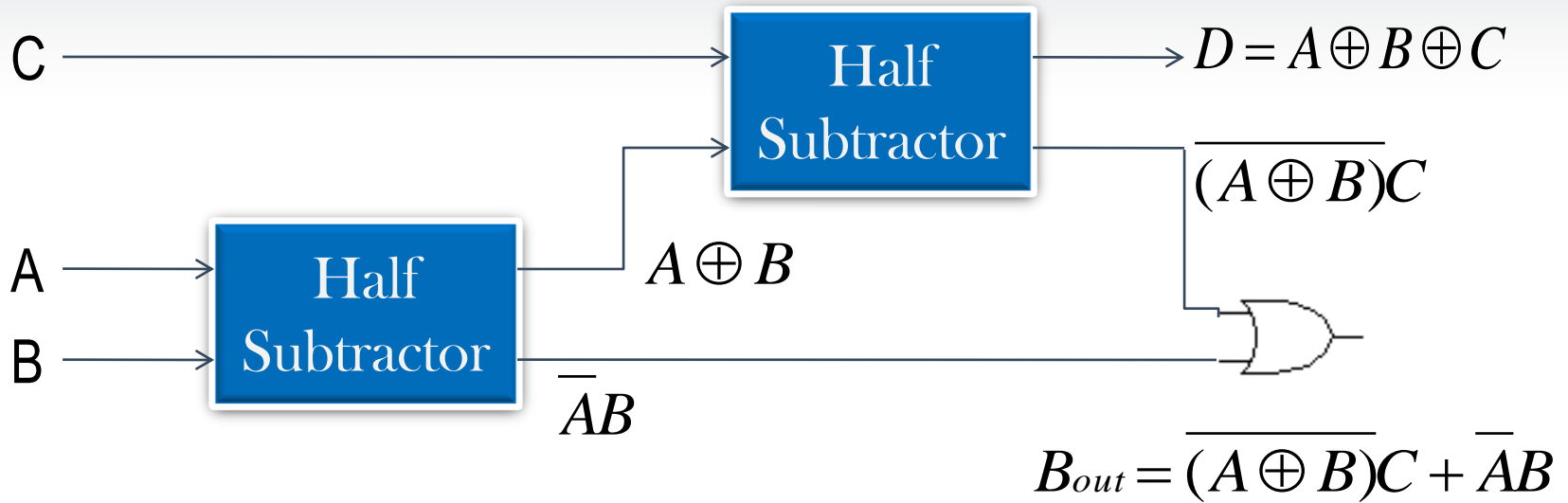


$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = (A \oplus B)C_{in} + AB$$

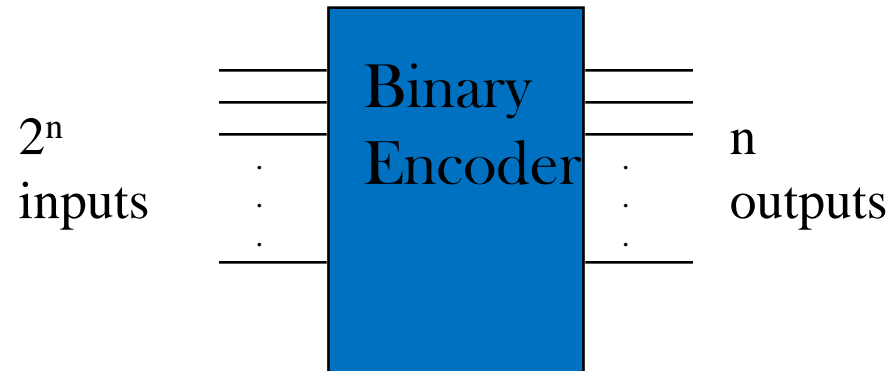


FULL SUBTRACTOR USING HALF SUBTRACTORS





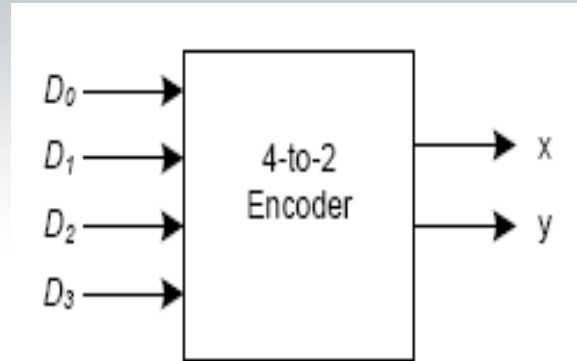
- It receives 2^n inputs and outputs a n bit binary value corresponding to the one input that has a value of 1.
- Only one input will be active at a time
- Useful for compressing data
- Can be developed using AND/OR gates
- are used in various components such as keyboards



4 X 2 ENCODER



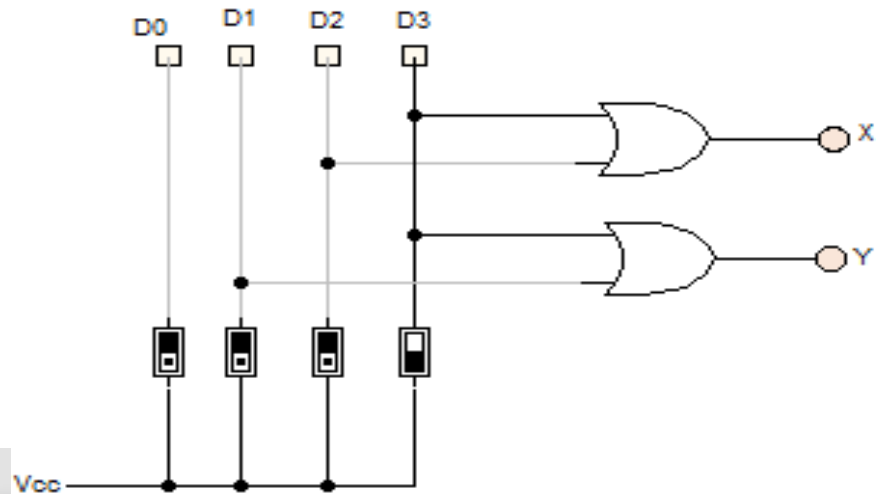
D_0	D_1	D_2	D_3	x	y
1	0	0	0	0	0
0	1	0	0	0	1
0	0	1	0	1	0
0	0	0	1	1	1



From the truth table, we can express the outputs as:

$$x = D_2 + D_3$$

$$y = D_1 + D_3$$



8 X 3 ENCODER



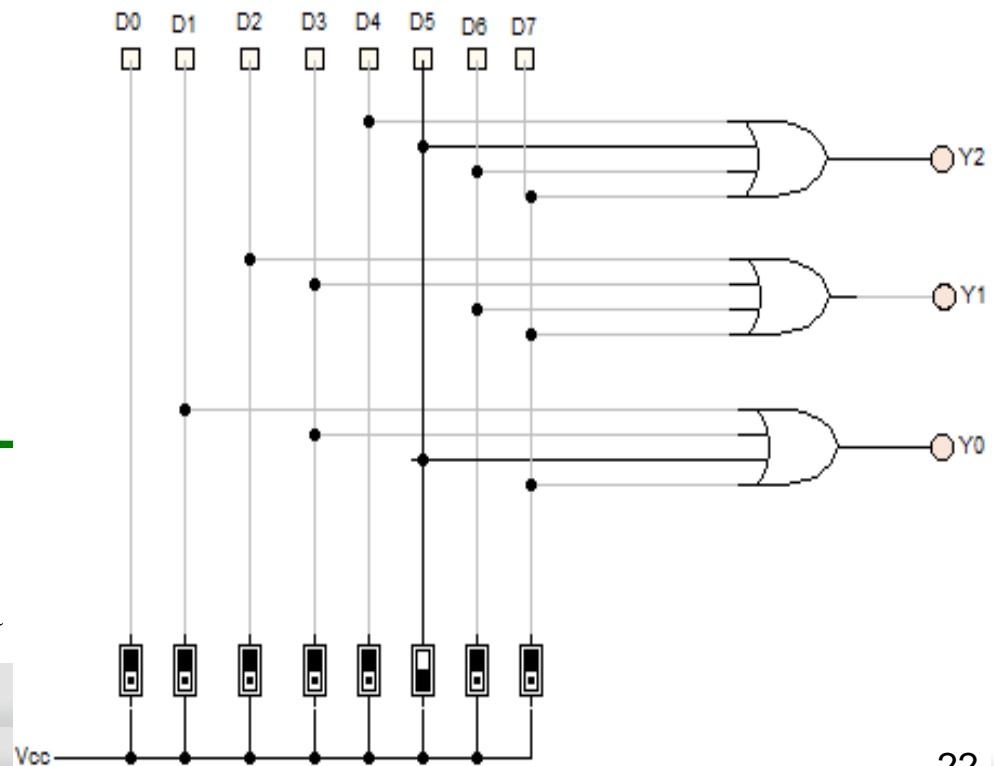
Inputs								Outputs		
D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇	y ₂	y ₁	y ₀
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	0
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	0
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	0

From the truth table, we can express the outputs as:

$$y_2 = D_4 + D_5 + D_6 + D_7$$

$$y_1 = D_2 + D_3 + D_6 + D_7$$

$$y_0 = D_1 + D_3 + D_5 + D_7$$



At any one time,
only
one input line has a
value of 1.





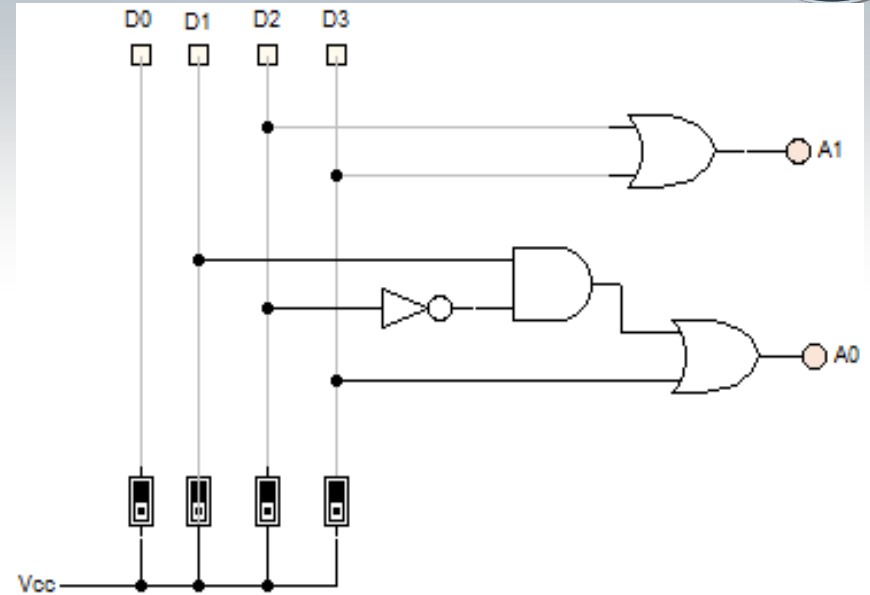
- A priority encoder works just a regular encoder, with one exception: whenever one or more input is active, the output is set to correspond to the highest active input
- Assign priorities to the inputs
- When more than one input are asserted, the output generates the code of the input with the highest priority
- For example, in a 4-to-2 encoder, if inputs 0,1 and 3 are active, then the $y_1 y_0 = 1 1$ output is set, corresponding to the input 3.



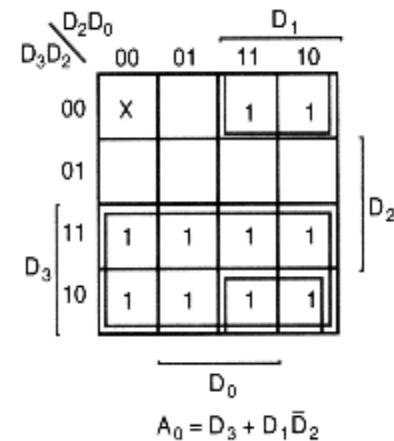
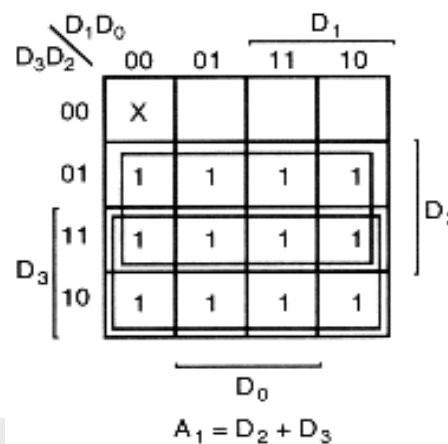
4 X 2 PRIORITY ENCODER



Inputs				Outputs	
D ₃	D ₂	D ₁	D ₀	A ₁	A ₀
0	0	0	0	X	X
0	0	0	1	0	0
0	0	1	X	0	1
0	1	X	X	1	0
1	X	X	X	1	1



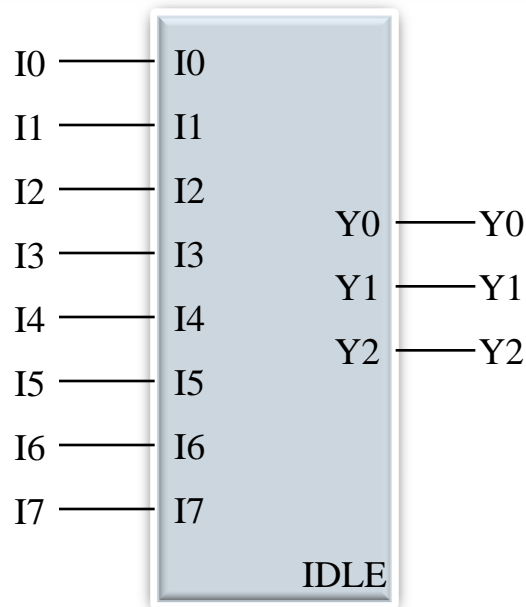
This can be minimised using the K-map as follows:



8-TO-3 PRIORITY ENCODER



- What if more than one input line has a value of 1?
- Ignore “lower priority” inputs.
- Idle indicates that no input is a 1.
- Priority encoders rank inputs and encode the highest priority input



Inputs								Outputs			
I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7	y_2	y_1	y_0	Idle
0	0	0	0	0	0	0	0	x	x	x	1
1	0	0	0	0	0	0	0	0	0	0	0
X	1	0	0	0	0	0	0	0	0	1	0
X	X	1	0	0	0	0	0	0	1	0	0
X	X	X	1	0	0	0	0	0	0	1	0
X	X	X	X	1	0	0	0	1	0	0	0
X	X	X	X	X	1	0	0	1	0	1	0
X	X	X	X	X	X	1	0	1	1	0	0
X	X	X	X	X	X	X	1	1	1	1	0



8-TO-3 PRIORITY ENCODER



Priority Encoder :

$$H7 = I7 \text{ (Highest Priority)}$$

$$H6 = I6 \cdot I7'$$

$$H5 = I5 \cdot I6' \cdot I7'$$

$$H4 = I4 \cdot I5' \cdot I6' \cdot I7'$$

$$H3 = I3 \cdot I4' \cdot I5' \cdot I6' \cdot I7'$$

$$H2 = I2 \cdot I3' \cdot I4' \cdot I5' \cdot I6' \cdot I7'$$

$$H1 = I1 \cdot I2' \cdot I3' \cdot I4' \cdot I5' \cdot I6' \cdot I7'$$

$$H0 = I0 \cdot I1' \cdot I2' \cdot I3' \cdot I4' \cdot I5' \cdot I6' \cdot I7'$$

$$IDLE = I0' \cdot I1' \cdot I2' \cdot I3' \cdot I4' \cdot I5' \cdot I6' \cdot I7'$$

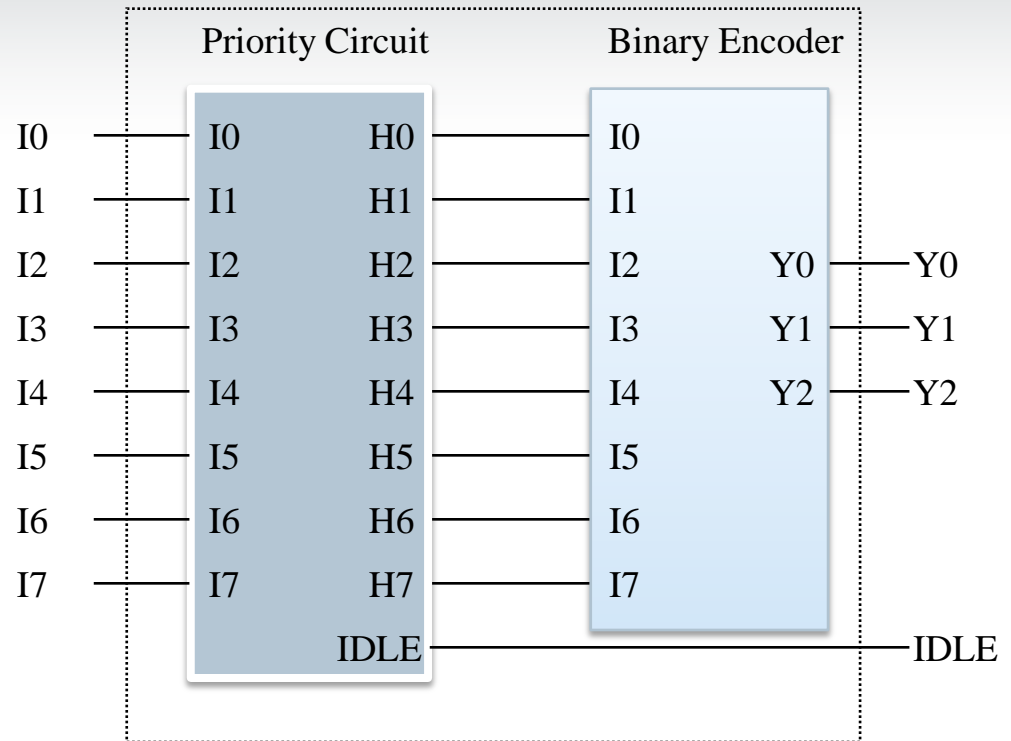
Encoder

$$Y0 = I1 + I3 + I5 + I7$$

$$Y1 = I2 + I3 + I6 + I7$$

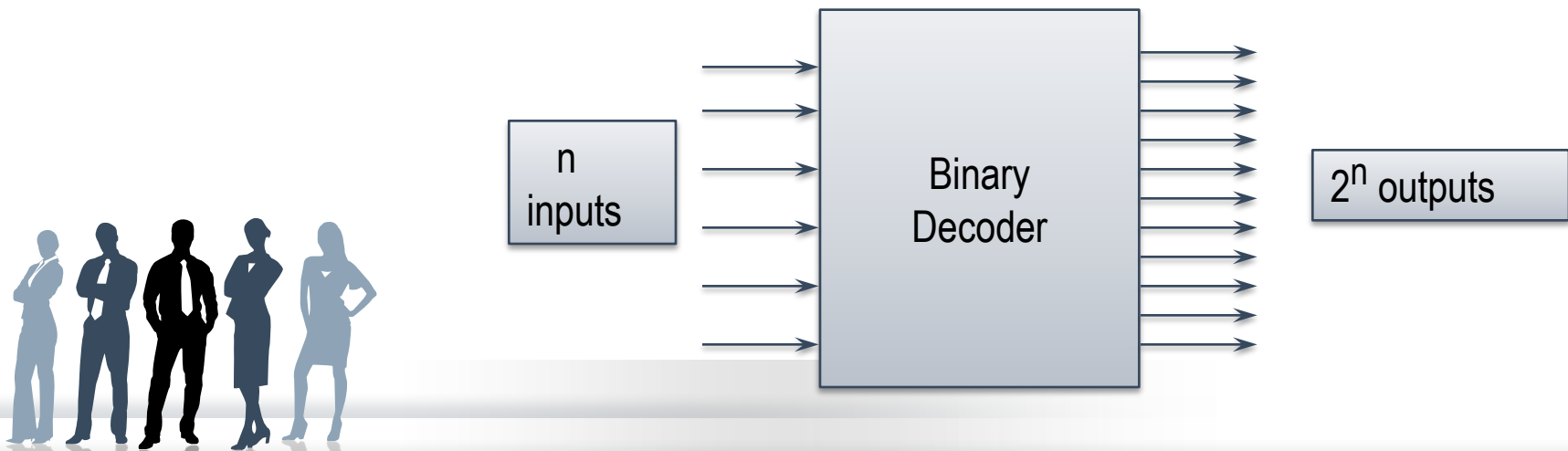
$$Y2 = I4 + I5 + I6 + I7$$

Priority Encoder

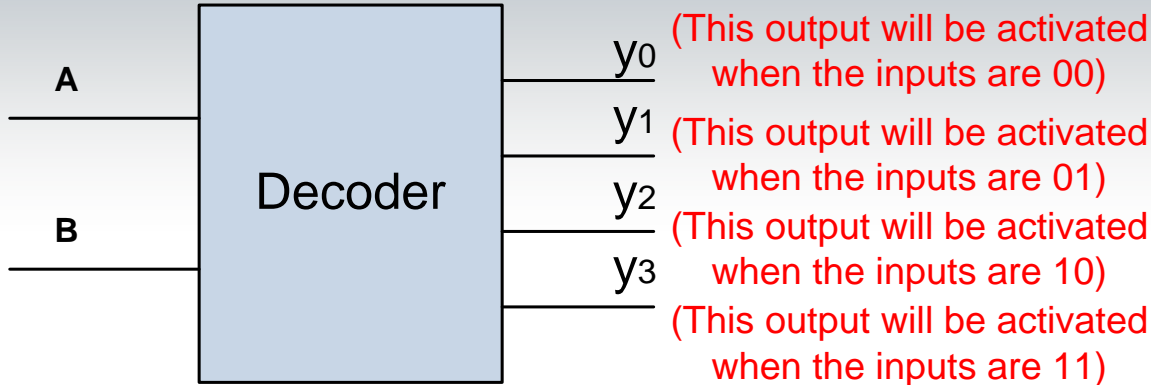




- A decoder accepts a binary value as input and decodes it.
- It has n inputs and 2^n outputs, numbered from 0 to $2^n - 1$.
- Each output represents one minterm of the inputs
- The output corresponding to the value of the n inputs is activated
- Only one output is a 1 for any given input
- For example, a decoder with three inputs and eight outputs will activate output 6 whenever the input values are 110.

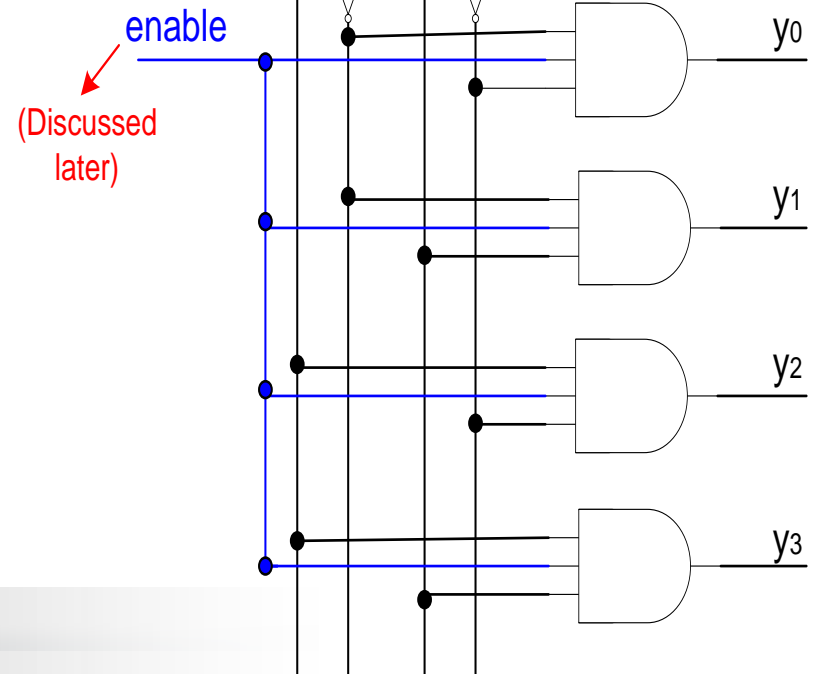


2 X 4 DECODER



(This output will be activated when the inputs are 00)
 (This output will be activated when the inputs are 01)
 (This output will be activated when the inputs are 10)
 (This output will be activated when the inputs are 11)

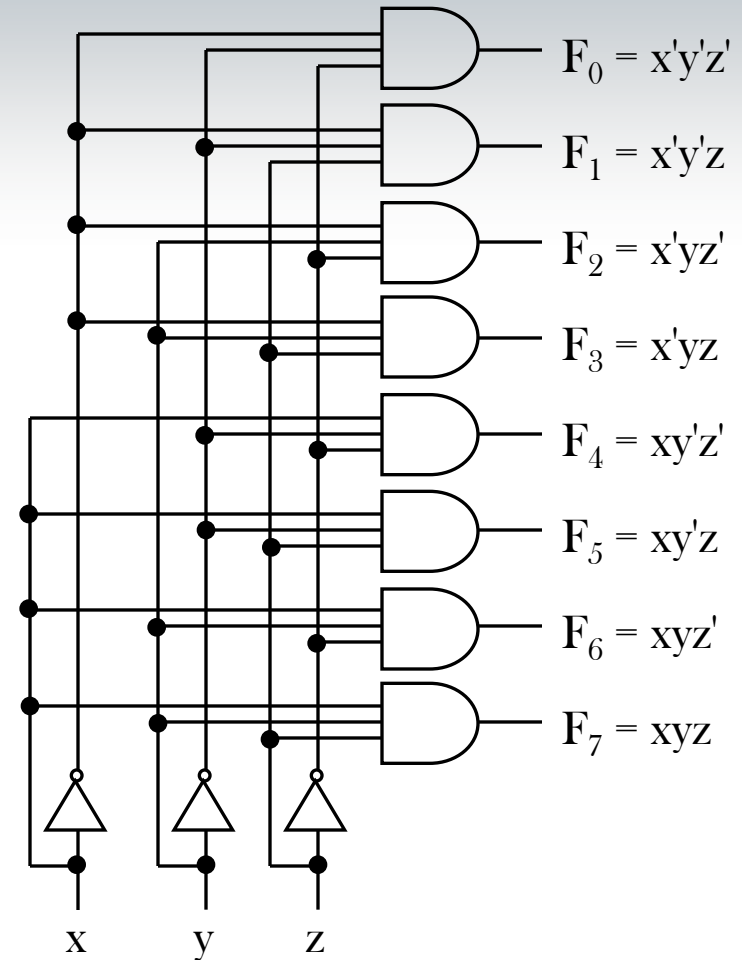
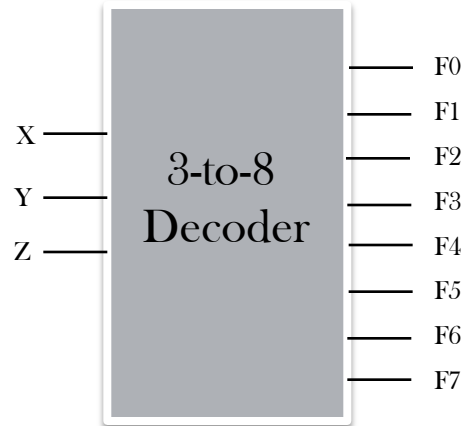
en	A	B	y_0	y_1	y_2	y_3
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1
0	-	-	0	0	0	0



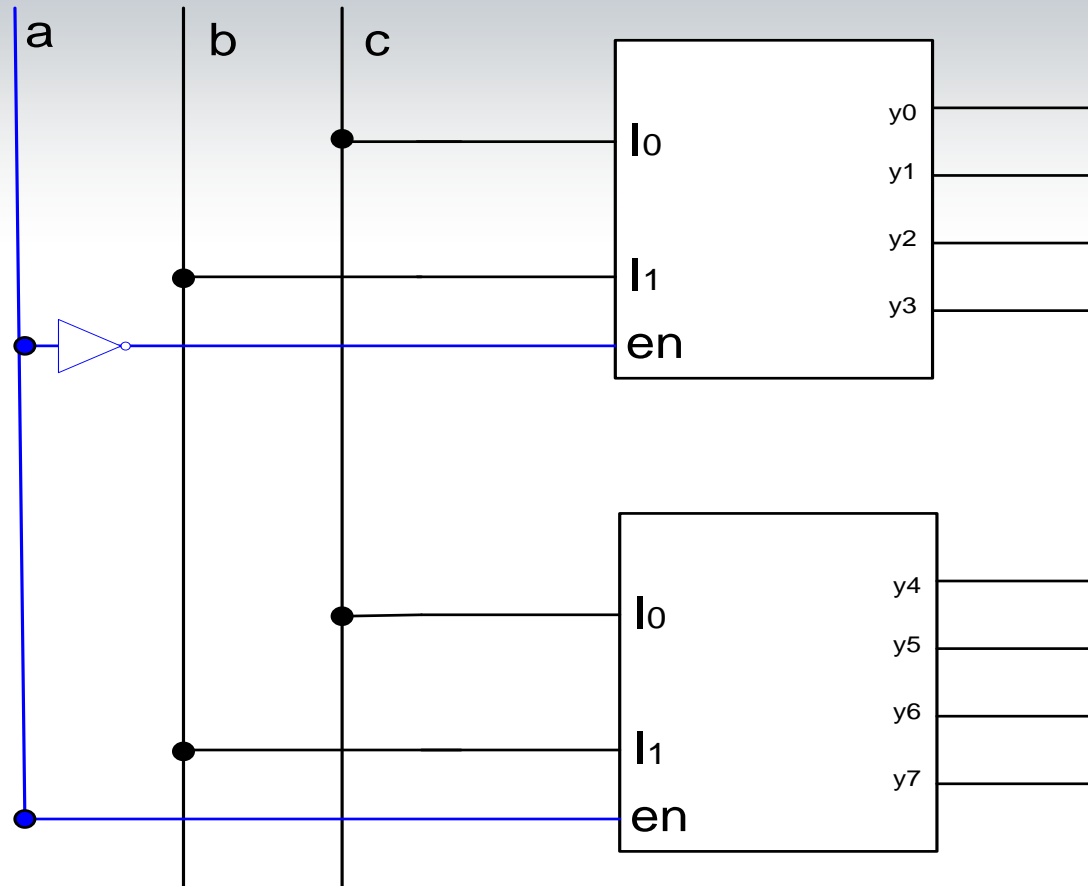
3 X 8 DECODER



x	y	z	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1



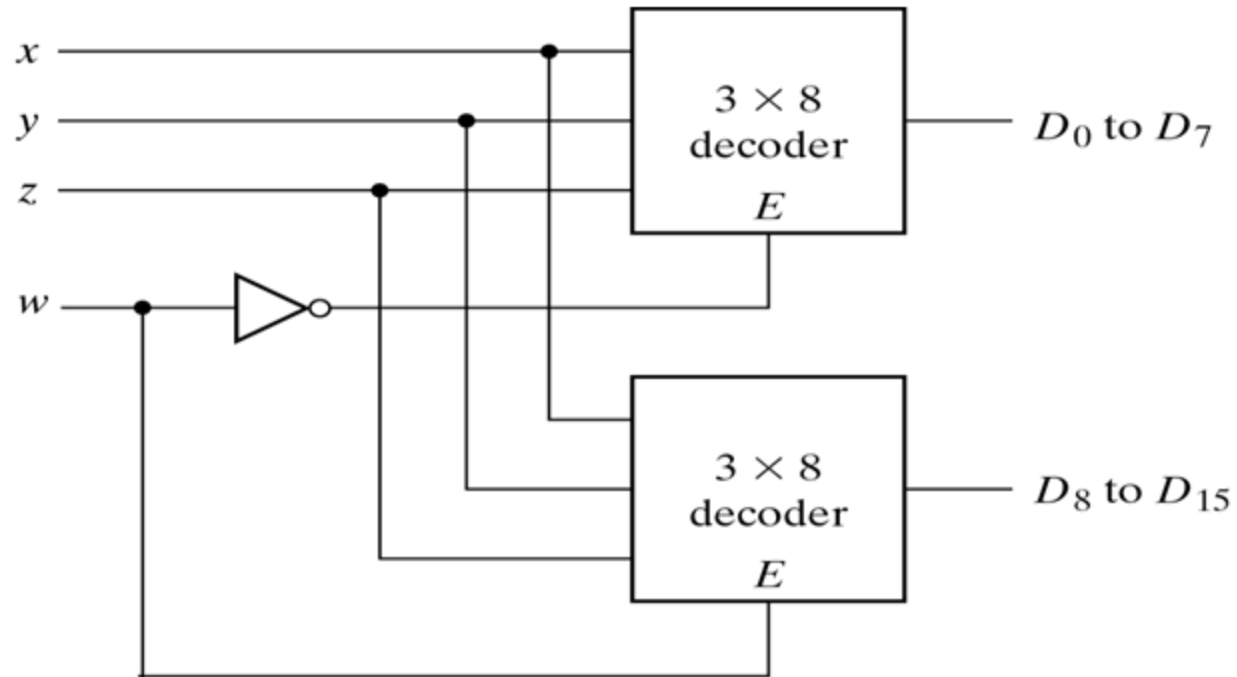
CONSTRUCTING 3 X 8 USING 2 X 4 DECODER



3 TO 8 DECODERS TO MAKE 4 TO 16 DECODER



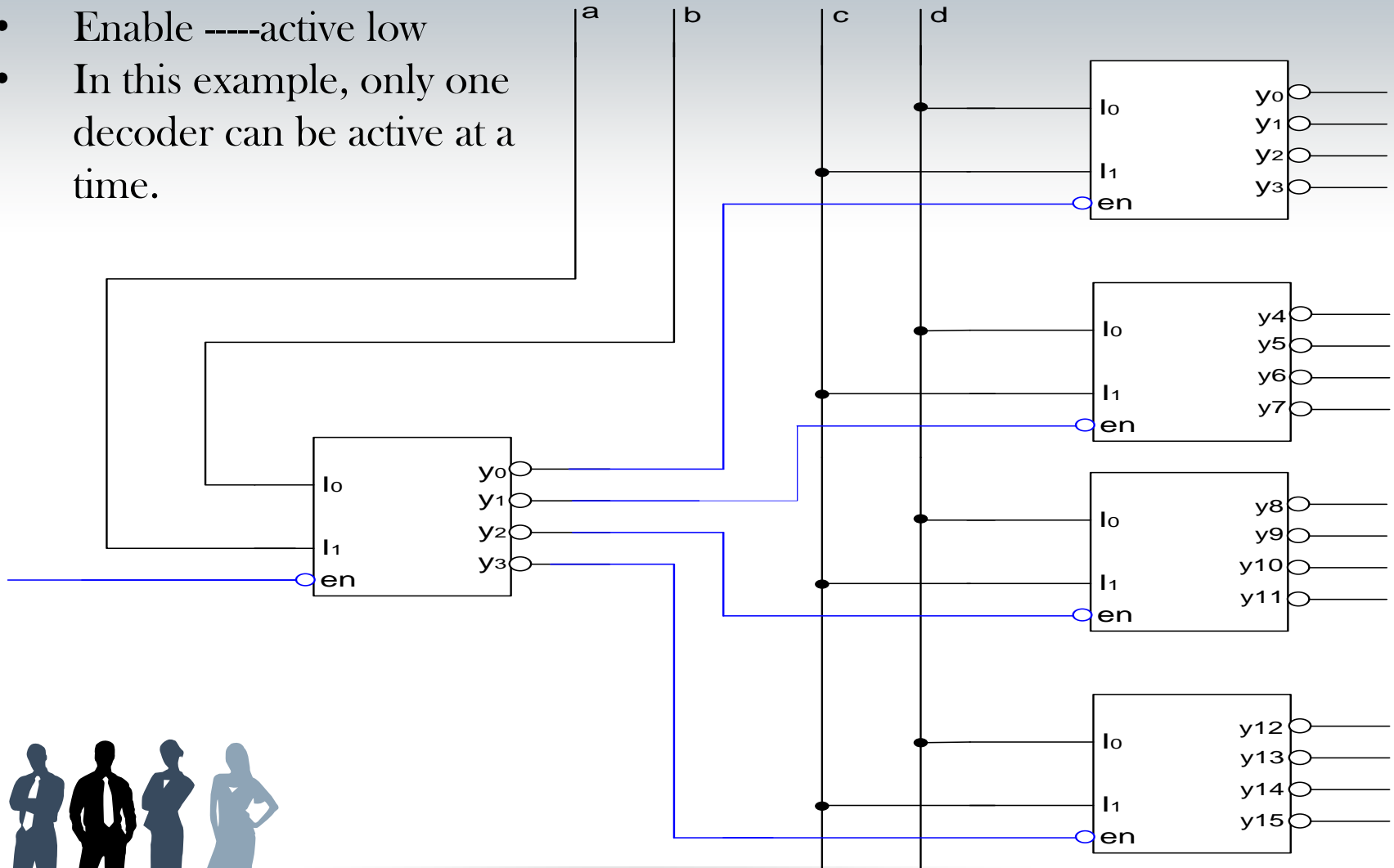
- Enable ----active high
- In this example, only one decoder can be active at a time.
- x, y, z ----input
- w ----strobe or enable



CONSTRUCTING 4 X 16 USING 2 X 4 DECODER



- Enable -----active low
- In this example, only one decoder can be active at a time.



IMPLEMENTING FUNCTIONS USING DECODERS



- Any n -variable logic function can be implemented using a single n -to- 2^n decoder to generate the minterms
 - OR gate forms the sum.
 - The output lines of the decoder corresponding to the minterms of the function are used as inputs to the or gate.
- Any combinational circuit with n inputs and m outputs can be implemented with an n -to- 2^n decoder with m OR gates.
- Suitable when a circuit has many outputs, and each output function is expressed with few minterms



IMPLEMENTING FUNCTIONS USING DECODERS

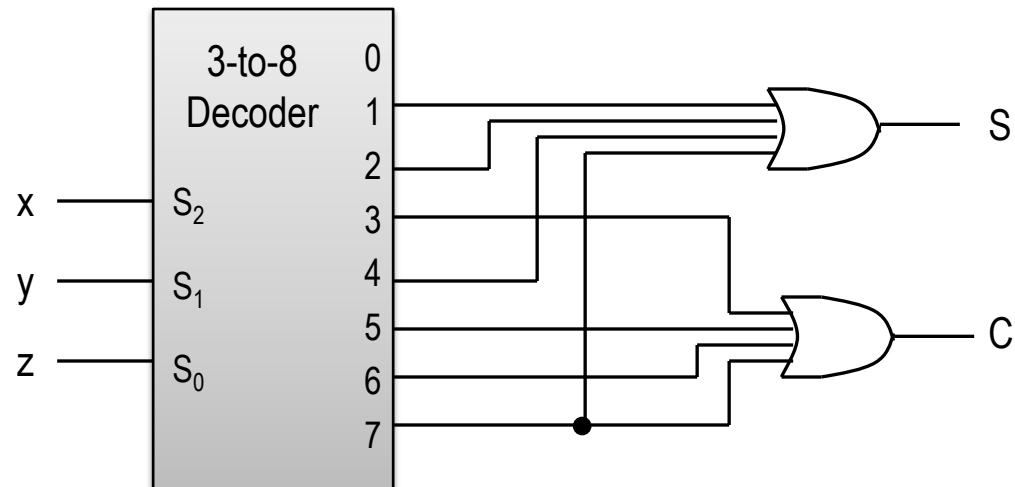


Example: Full adder

$$S(x, y, z) = S(1, 2, 4, 7)$$

$$C(x, y, z) = S(3, 5, 6, 7)$$

x	y	z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



BCD-TO-SEVEN-SEGMENT DECODER



- Digital readouts on many digital products often use LED seven-segment displays.
- Each digit is created by lighting the appropriate segments. The segments are labeled a,b,c,d,e,f,g
- The decoder takes a BCD input and outputs the correct code for the seven-segment display.
- Input: A 4-bit binary value that is a BCD coded input.
- Outputs: 7 bits, a through g for each of the segments of the display.
- Operation: Decode the input to activate the correct segments



(a) Segment designation



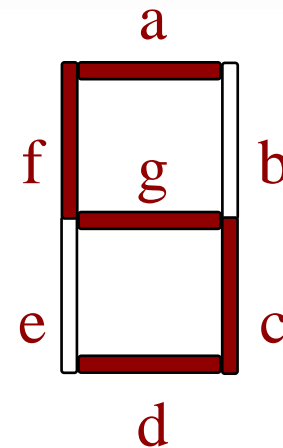
(b) Numeric designation for display



LISTING THE SEGMENTS



- 0 a,b,c,d,e,f
- 1 b,c
- 2 a,b,d,e,g
- 3 a,b,c,d,g
- 4 b,c,f,g
- 5 a,c,d,f,g
- 6 a,c,d,e,f,g
- 7 a,b,c
- 8 a,b,c,d,e,f,g
- 9 a,b,c,d,f,g



TRUTH TABLE FOR BCD-TO-SEVEN-SEGMENT DECODER



BCD Input				Seven Segment Decoder						
A	B	C	D	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
all other inputs				x	x	x	x	x	x	x

- Fill in don't cares for *undefined* outputs.
 - Leads to a reduced implementation
- For these combinations of *undefined* inputs never to happen, fill '0'





For segment "a" :

		CD			
		00	01	11	10
AB	00	1	0	1	1
	01	0	1	1	1
	11	X	X	X	X
	10	1	1	X	X

$$F_a = A + C + \overline{B}\overline{D} + BD$$

		CD			
		00	01	11	10
AB	00	1	0	1	1
	01	0	1	1	1
	11	X	X	X	X
	10	1	1	X	X

$$F_{a2} = A$$

		CD			
		00	01	11	10
AB	00	1	0	1	1
	01	0	1	1	1
	11	X	X	X	X
	10	1	1	X	X

$$F_{a4} = BD$$

		CD			
		00	01	11	10
AB	00	1	0	1	1
	01	0	1	1	1
	11	X	X	X	X
	10	1	1	X	X

$$F_{a1} = C$$

		CD			
		00	01	11	10
AB	00	1	0	1	1
	01	0	1	1	1
	11	X	X	X	X
	10	1	1	X	X

$$F_{a3} = \overline{B}\overline{D}$$

Similarly do for segment b, c, d, e, f and g.
Then draw logical diagram



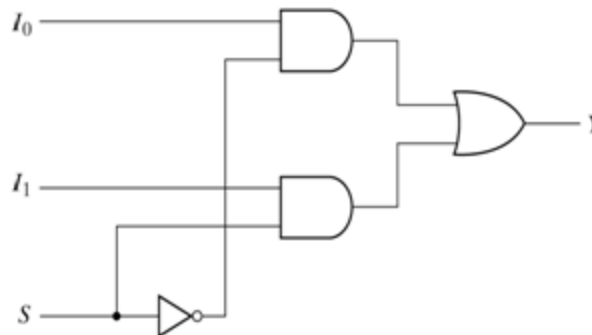


- It is a selector, it chooses one of its data inputs and passes it to the output according to some other selection inputs
- Select an input value with one or more select bits
- Use for transmitting data
- Allows for conditional transfer of data
- Sometimes called a mux
- Consider four binary data inputs as inputs of a multiplexer. Two select signals will determine which of the four inputs will be passed to the output.

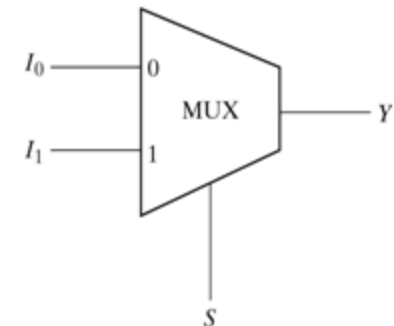
For 2 to 1 MUX

- 2 Inputs I_0 AND I_1
- Select line S
- Output Y

S	Y
0	I_0
1	I_1



(a) Logic diagram



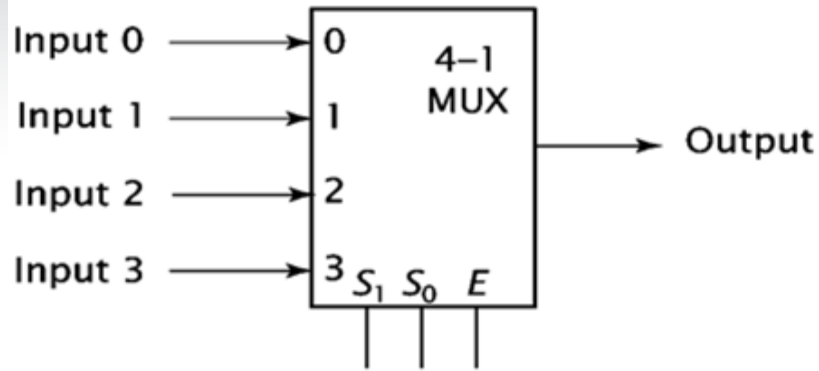
(b) Block diagram

$$Y = \bar{S}I_0 + SI_1$$

2-to-1-Line Multiplexer

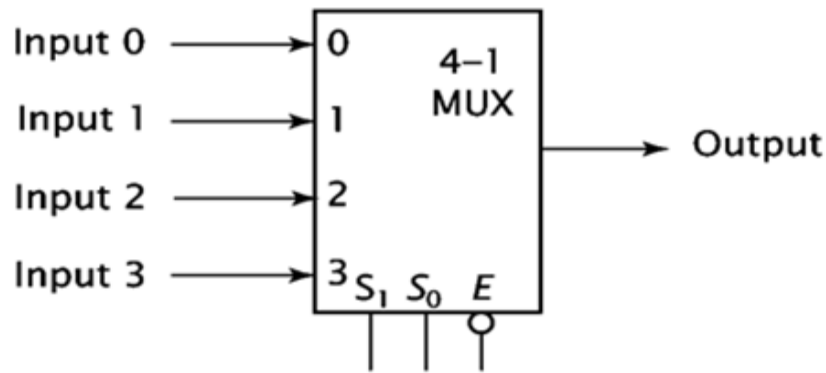


4 TO 1 MULTIPLEXER



S_1	S_0	E	Output
X	X	0	Z
0	0	1	Input 0
0	1	1	Input 1
1	0	1	Input 2
1	1	1	Input 3

Multiplexer schematic representation with active high enable signal

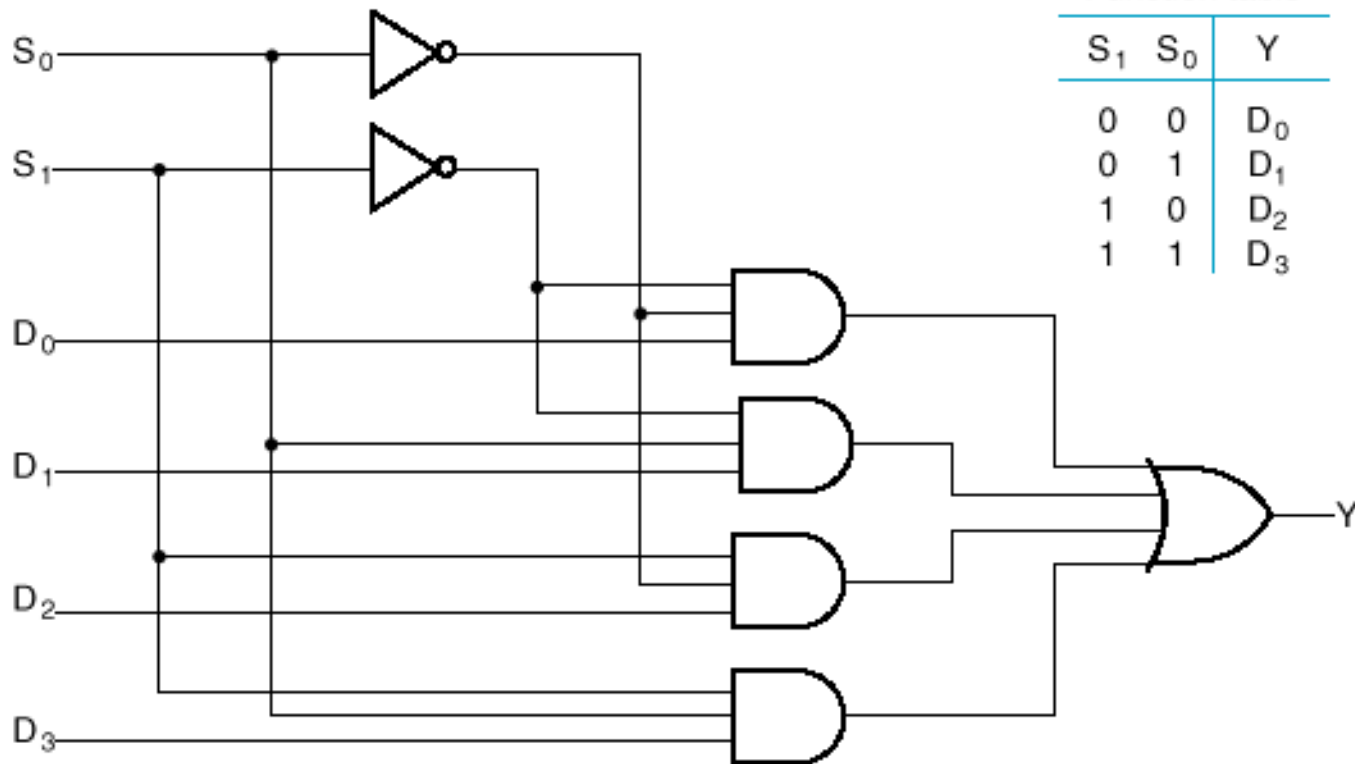


S_1	S_0	E	Output
X	X	1	Z
0	0	0	Input 0
0	1	0	Input 1
1	0	0	Input 2
1	1	0	Input 3

Multiplexer schematic representation with active low enable signal



4 TO 1 MULTIPLEXER



Function table

S ₁	S ₀	Y
0	0	D ₀
0	1	D ₁
1	0	D ₂
1	1	D ₃

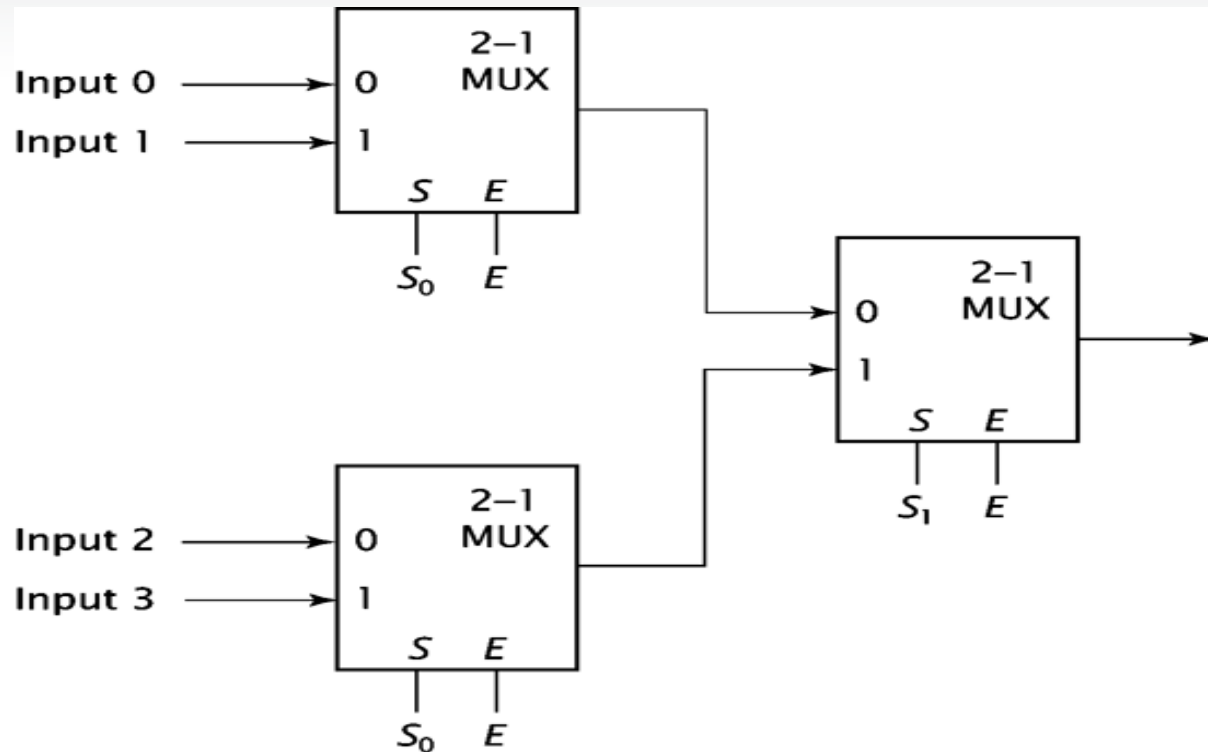


$$Y = \overline{S_1} \overline{S_0} D_0 + \overline{S_1} S_0 D_1 + S_1 \overline{S_0} D_2 + S_1 S_0 D_3$$

CONSTRUCTION OF 4 TO 1 USING 2 TO 1 MUX



- Multiplexers can be cascaded to select from a large number of inputs
- 4 to 1 multiplexer made of 2 to 1 multiplexers



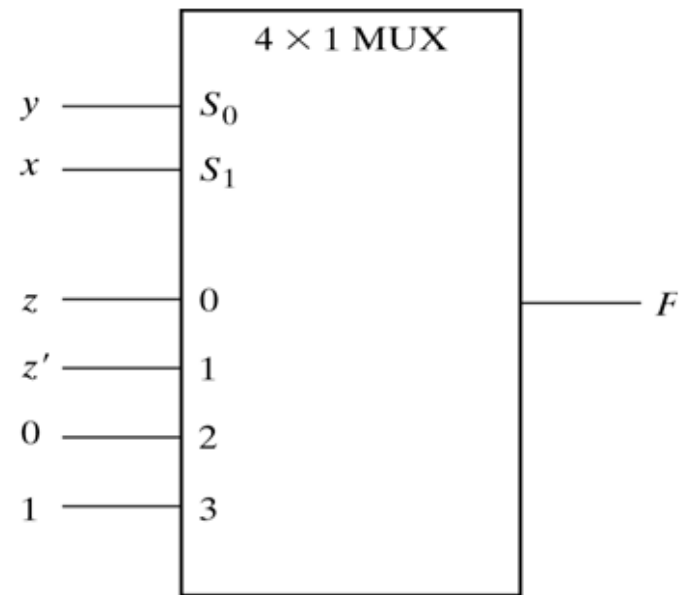
IMPLEMENTING BOOLEAN FUNCTION WITH MUX



- Connect input variables to select inputs of multiplexer ($n-1$ for n variables)
- Set data inputs to multiplexer equal to values of function for corresponding assignment of select variables
- Using a variable at data inputs reduces size of the multiplexer

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

(a) Truth table



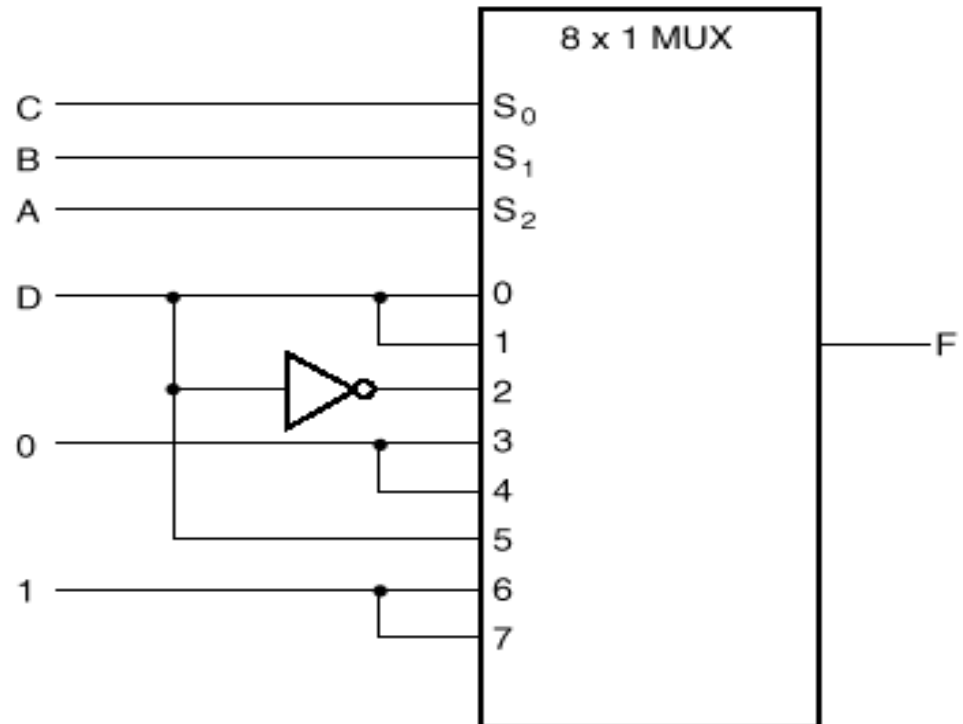
(b) Multiplexer implementation



IMPLEMENTING A FOUR- INPUT FUNCTION WITH A MULTIPLEXER

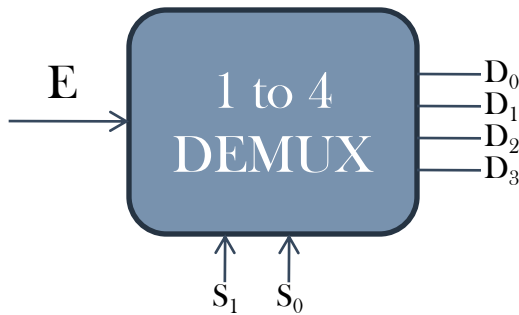


A	B	C	D	F	
0	0	0	0	0	$F = D$
0	0	0	1	1	
0	0	1	0	0	$F = D$
0	0	1	1	1	
0	1	0	0	1	$F = \bar{D}$
0	1	0	1	0	
0	1	1	0	0	$F = 0$
0	1	1	1	0	
1	0	0	0	0	$F = 0$
1	0	0	1	0	
1	0	1	0	0	$F = D$
1	0	1	1	1	
1	1	0	0	1	$F = 1$
1	1	0	1	1	
1	1	1	0	1	$F = 1$
1	1	1	1	1	

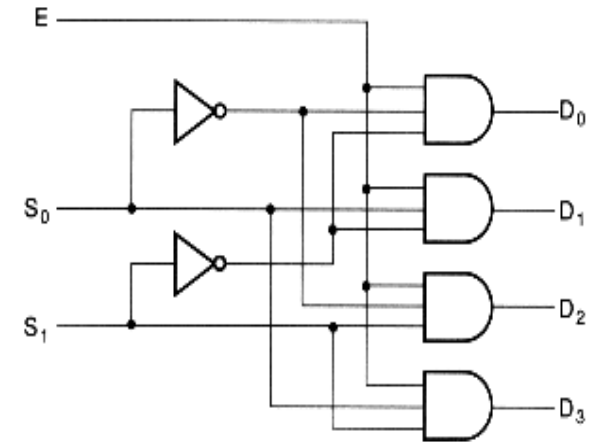




- The de-multiplexer performs the inverse function of a multiplexer
- It receives information on one line and transmits it onto one of 2^n possible output lines.
- The selection is by n input select lines
- Note that a one to four multiplexer is really a two to four decoder with an additional enable input E which is the input data line.



S_1	S_0	D_0	D_1	D_2	D_3
0	0	E	0	0	0
0	1	0	E	0	0
1	0	0	0	E	0
1	1	0	0	0	E



$$D_0 = \overline{S_1} \overline{S_0} E \quad D_2 = S_1 \overline{S_0} E$$

$$D_1 = \overline{S_1} S_0 E \quad D_3 = S_1 S_0 E$$

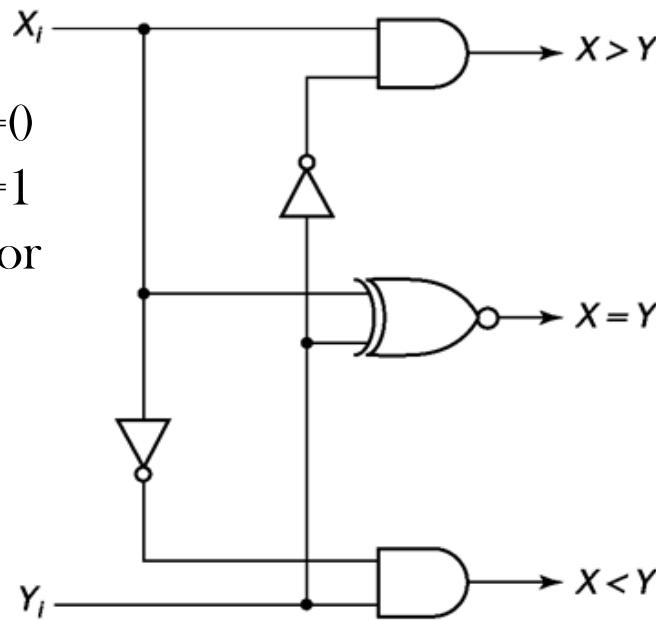


COMPARATORS



- A comparator compares a two n-bit binary values to determine which is greater or if they are equal
- Consider the simple 1-bit comparator to illustrate the design
- It is possible to extend the design for multi-bit numbers

- $X > Y$ only if $X_i = 1, Y_i = 0$
- $X < Y$ only if $X_i = 0, Y_i = 1$
- $X = Y$ only if $X_i = Y_i = 0$ or $X_i = Y_i = 1$



X_i	Y_i	$X > Y$	$X = Y$	$X < Y$
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0

(b)

(a)



BCD TO EXCESS 3 CODE CONVERTER



- BCD is a code for the decimal digits 0-9
- Excess-3 is also a code for the decimal digits
- Inputs: a BCD input, A,B,C,D with A as the most significant bit and D as the least significant bit.
- Outputs: an Excess-3 output W,X,Y,Z that corresponds to the BCD input.
- Internal operation - circuit to do the conversion in combinational logic.
- Excess-3 code is easily formed by adding a binary 3 to the binary or BCD for the digit.
- There are 16 possible inputs for both BCD and Excess-3.
- It can be assumed that only valid BCD inputs will appear so the six combinations not used can be treated as don't cares.



BCD TO EXCESS 3 CODE CONVERTER



Decimal	BCD input				Excess-3 output			
	A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0



REDUCTION USING K-MAP



AB \ CD		C			
		00	01	11	10
A	00				
	01		1	1	1
	11	X	X	X	X
	10	1	1	X	X

$$W = A + BC + BD$$

AB \ CD		C			
		00	01	11	10
A	00		1	1	1
	01	1			
	11	X	X	X	X
	10		1	X	X

$$X = \bar{B}C + \bar{B}D + B\bar{C}\bar{D}$$

AB \ CD		C			
		00	01	11	10
A	00	1		1	
	01	1		1	
	11	X	X	X	X
	10	1		X	X

$$Y = CD + \bar{C}\bar{D}$$

AB \ CD		C			
		00	01	11	10
A	00	1			1
	01	1			1
	11	X	X	X	X
	10	1		X	X

$$Z = \bar{B}$$



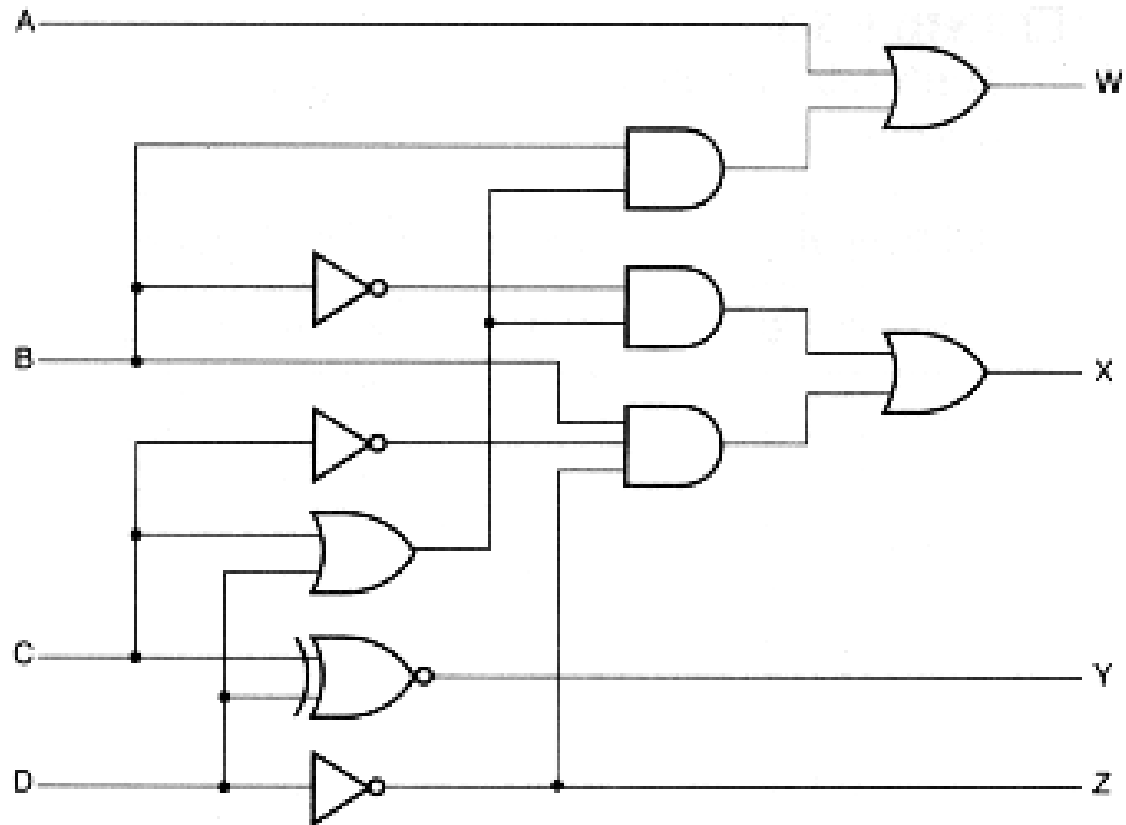


$$W = A + BC + BD = A + B(C + D)$$

$$X = \bar{B}C + \bar{B}D + B\bar{C}\bar{D} = \bar{B}(C + D) + B\bar{C}\bar{D}$$

$$Y = CD + \bar{C}\bar{D} = \overline{C \oplus D}$$

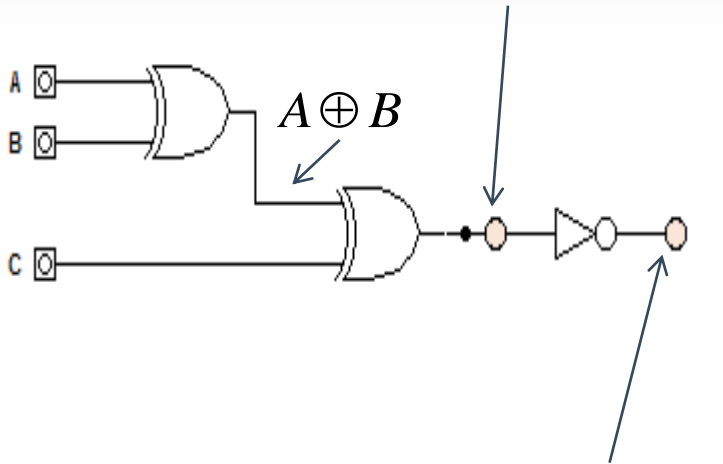
$$Z = \bar{D}$$



EVEN & ODD PARITY GENERATOR FOR 3 BITS



$$EP = A \oplus B \oplus C$$



$$OP = \overline{A \oplus B \oplus C}$$

A	B	C	EP (Even Parity)	OP (Odd Parity)
0	0	0	0	1
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0



BCD TO GRAY CONVERTER



Decimal	BCD Input				GRAY Output			
	A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	0	0	0	1	1
3	0	0	1	1	0	0	1	0
4	0	1	0	0	0	1	1	0
5	0	1	0	1	0	1	1	1
6	0	1	1	0	0	1	0	1
7	0	1	1	1	0	1	0	0
8	1	0	0	0	1	1	0	0
9	1	0	0	1	1	1	0	1
10-15	all other inputs				X	X	X	X



WRITING EQUATIONS AND REDUCTIONS USING K-MAP



		CD			
		00	01	11	10
AB	00	0	0	0	0
	01	0	0	0	0
	11		X	X	X
	10	X	1	X	X

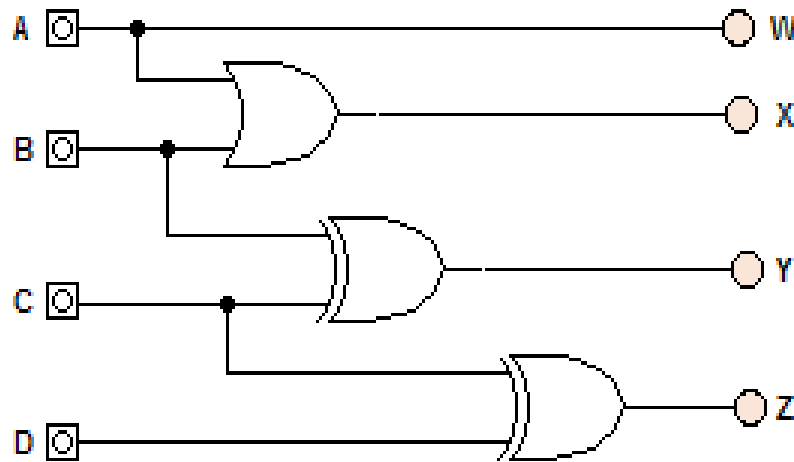
$$W = A$$

		CD			
		00	01	11	10
AB	00	0	0	0	0
	01	1	1	1	1
	11	X	X	X	X
	10	1	1	X	X

$$X = A + B$$

		CD			
		00	01	11	10
AB	00	0	0	1	1
	01	1	1	0	0
	11	X	X	X	X
	10	0	0	X	X

$$Y = B\bar{C} + \bar{B}C = B \oplus C$$



		CD			
		00	01	11	10
AB	00	0	1	0	1
	01	0	1	0	1
	11	X	X	X	X
	10	0	1	X	X

$$Z = \bar{C}D + C\bar{D} = C \oplus D$$



