

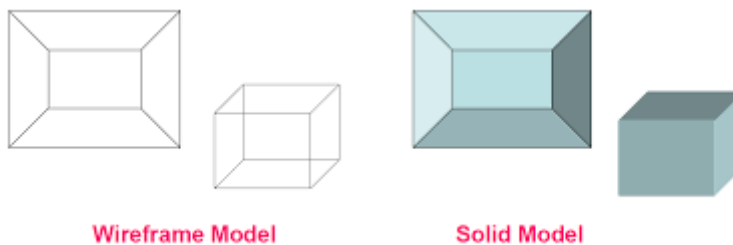
## Unit 6

### Solid Modeling

There are three types of 3D model:

1. **Wireframe** - Uses lines, arcs and curves, as if the object shape were made with pieces of wire.
2. **Surface** - Uses surfaces to show the outside of the model, as if it were made from pieces of stretchy paper. It's hollow inside, but looks realistic on the outside.
3. **Solid** - Uses solid material throughout, not hollow.

Solid model of an object is a more complete representation than its surface (wireframe) model. It provides more topological information in addition to the geometrical information which helps to represent the solid unambiguously.



#### Geometry vs topology:

**Geometry:** Metrics and dimensions of the solid object. Location of the object in a chosen coordinate system.

**Topology:** Combinational information like connectivity, associativity, and neighborhood information. Invisible relationship information.

#### Solid Modeling

Solid modeling is a consistent set of principles for mathematical and computer modeling of three-dimensional solids.

- A mathematical technique for representing solid objects.
- An object with following specification is called solid:  
Vertices, edges, surfaces, weight, and volume.

Solid modeling is based on **complete, valid and unambiguous** geometric representation of physical object.

- **Complete:** points in space can be classified. (inside/outside).
- **Valid:** Vertices, edges, faces are connected properly.
- **Unambiguous:** there can be only one interpretation of object.

## Advantages of solid Models

Unlike wireframe and surface representations which contain only geometrical data, the solid model uses topological information in addition to the geometrical information to represent the object unambiguously and completely. Solid models results in accurate design, helps to further the goal of CAD/CAM like CIM. Flexible manufacturing leading to better automation of the manufacturing process.

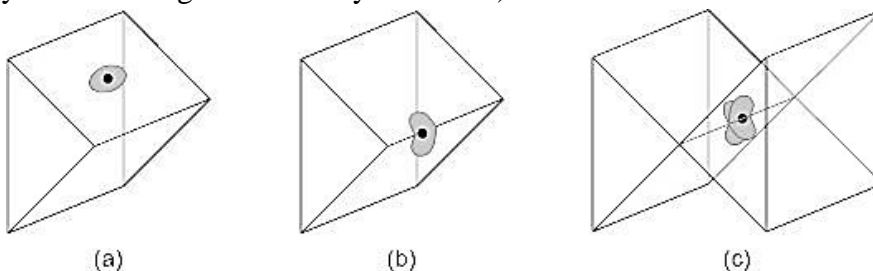
### The common representations in solid modeling are:

1. **Spatial Enumeration:** In this simplest form of 3D volumetric raster model, a section of 3D space is described by a matrix of evenly spaced cubic volume elements called voxels.
2. **Cell Decomposition:** This is a hierarchical adaptation of spatial enumeration. 3D space is sub-divided into cells. Cells could be of different sizes. These simple cells are glued together to describe a solid object.
3. **Boundary Representation:** The solid is represented by its boundary which consists of a set of faces, a set of edges and a set of vertices as well as their topological relations.
4. **Sweep Methods:** In this technique a planar shape is moved along a curve. Translational sweep can be used to create prismatic objects and rotational sweep could be used for axisymmetric components.
5. **Primitive Instancing:** This modeling scheme provides a set of possible object shapes which are described by a set of parameters. Instances of object shape can be created by varying these parameters.
6. **Constructive Solid Geometry (CSG):** Primitive instances are combined using Boolean set operations to create complex objects

### Major modeling schemes:

#### ➤ Boundary representation:

- The solids are described by its boundary surface. Uses the description by vertices, edges and faces.
- The most common representation is the boundary polygons.
- Will be considered only with solid border 2-manifolds (wherever the point is each edge shared by two faces) i.e. the neighbors of any point of the border point are on a disk (that is to say that each edge is shared by two faces)



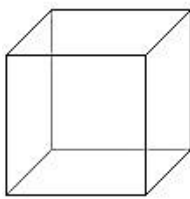
Figure(a) and (b) are *2-manifold* (c) is not *2-manifold*

**Polyhedron:**

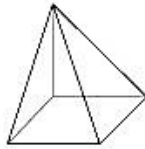
Solid delimited by a set of polygons whose edges belonging to two polygons (for solids 2-manifolds).

- Boundary representation for single polyhedron satisfy Euler's formula

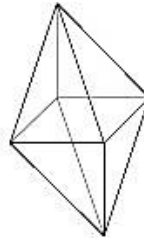
$$V-E+F=2 \quad \text{where, } V\text{- vertex, } E\text{-edges \& } F\text{- faces.}$$



$$\begin{aligned} V &= 8 \\ E &= 12 \\ F &= 6 \end{aligned}$$

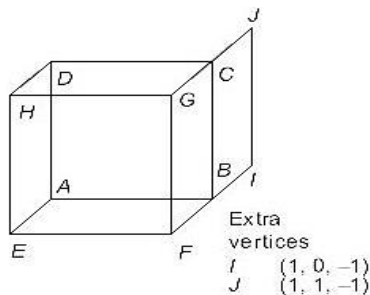


$$\begin{aligned} V &= 5 \\ E &= 8 \\ F &= 5 \end{aligned}$$



$$\begin{aligned} V &= 6 \\ E &= 12 \\ F &= 8 \end{aligned}$$

The **Euler's formula** is necessary but not sufficient to ensure that an object is a simple polyhedron.

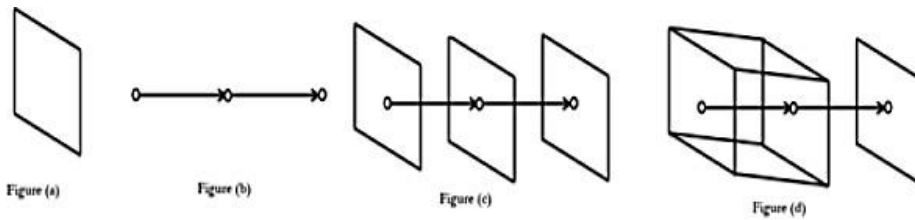


➤ **Spatial Partitioning Representation:**

- A solid is represented as a collection of adjoining non-intersecting object.
- For example: Octree

➤ **Sweep Representation:**

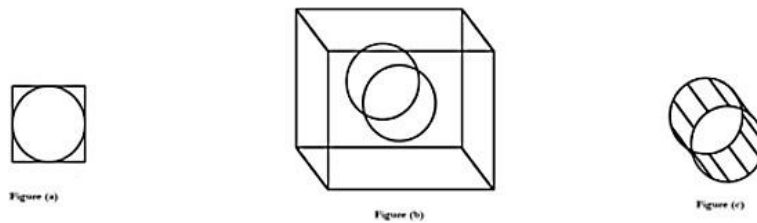
- Sweep representations create solid by moving a 2D shape (triangle, rectangle, polygon etc.) according to predefined rule (translating, rotating).
- Sweep representations are used to construct 3D object from 2D shape that have some kind of symmetry.
- For example, a prism can be generated using a translational sweep and rotational sweeps can be used to create curved surfaces like an ellipsoid or a torus.
- There are two types sweep representation:
  - **Translational Sweep:** A 2D shape is translated by a predefined translational vector. Example: following figure shows a translational sweep of rectangle. Steps:
    - Define a shape as a polygon vertex table as shown in figure (a).
    - Define a sweep path as a sequence of translation vectors figure (b).
    - Translate the shape; continue building a vertex table figure (c).
    - Define a surface table figure (d).



- **Rotational Sweep:** A 2D shape is rotated around a predefined rotational axis. Example: following figure shows a rotational sweep.

Steps:

- Define a shape as a polygon vertex table as shown in figure (a).
- Define a sweep path as a sequence of rotations.
- Rotate the shape; continue building a vertex table as shown in figure (b).
- Define a surface table as shown in figure (c).



### ➤ Constructive Solid Geometry (CSG):

A CSG model is based on the topological notion that a physical object can be divided into a set of primitives (basic elements or shapes) that can be combined in a certain order following a set of rules (Boolean operations) to form the object. Each primitive is bounded by a set of surfaces; usually closed and orientable.

- It is based on the idea of providing a set of predefined object types such as cubes, cone, sphere etc.
- A solid model is created by retrieving primitive solids and performing Boolean operations.
- Three types of Boolean operations:
  - Union (join): the operation combines two volumes included in the different solids into a single solid.
  - Subtract (cut): the operation subtracts the volume of one solid from the other solid object.
  - Intersection: the operation keeps only the volume common to both solids.

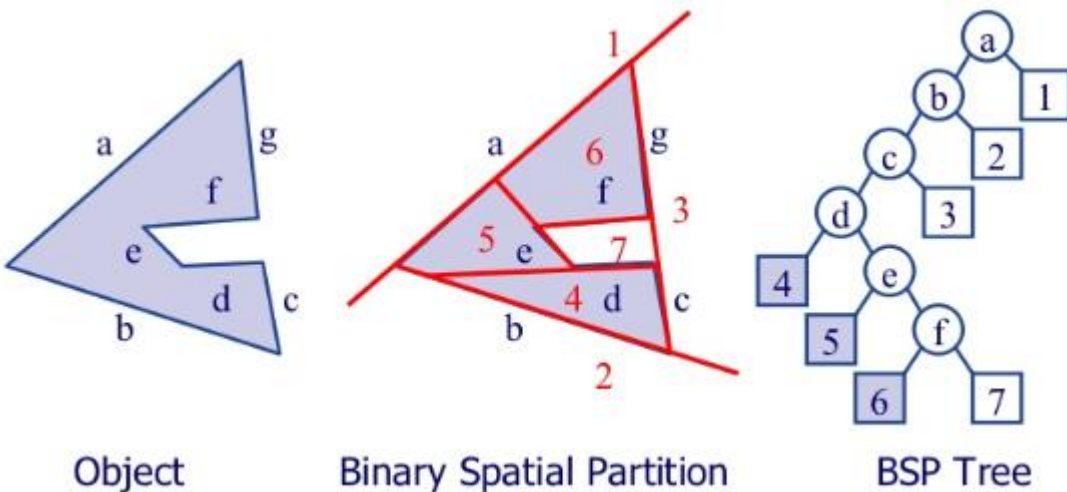
## Process of Making the Solid Models

To make the solid models you have to first make the wire frame model of the object and convert it into 3D view. Thereafter the surfaces are added to the 3D wire model to convert it into 3D solid model. For creating the solid models you need to have special CAD software that can create solid models. One of the most popular CAD software for solid modeling is SolidWorks. Its latest version is SolidWorks 2009. A number of other CAD software like AutoCAD and others also have features of creating the solid models.

## Binary Space Partitioning Tree (BSP)

Binary space partitioning is a generic process of recursively dividing a scene into two until the partitioning satisfies one or more requirements.

- At each step, the space is divided by a plane of arbitrary position and orientation.
- Each internal node of the tree is associated with a plane and has two child pointers (one for inside the polygon and the other to the outside).
- If the subspace is homogenous (fully indoors and outdoors), cease to be divided.



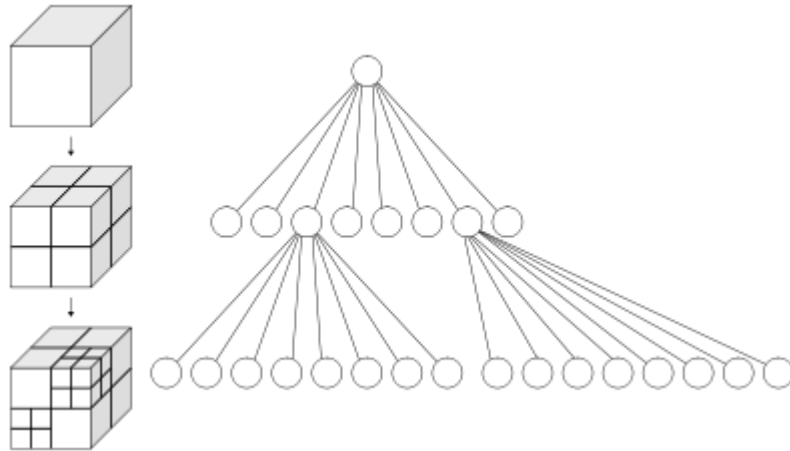
## **Octree Representation (Solid-object representation)**

This is the space-partitioning method for 3D solid object representation. Octrees are hierarchical tree structures that describes each region space as nodes. They are used to represent solid objects in some graphics system.

- Medical imaging and other applications that require displays of object cross sections commonly use octree representation. E.g. CT-scan.

Octrees are used to partition a 3D space by recursively subdividing it into eight octants. Octant subdivisions continue until the region of space contains only homogeneous octants.

- Octrees are often used in 3D graphics and 3D game engines.



***Fig: Left: Recursive subdivision of a cube into octants & Right: The corresponding octree***

### ***Advantages:***

- It provides a convenient representation for storing information about object interiors.
- They can represent arbitrary shapes and that we can quickly analyze what is present at a specific position in space.

### ***Disadvantages:***

- Imprecise representation, high storage demands and complex transformation operations.

**References**

- **Donald Hearne and M.Pauline Baker**, “Computer Graphics, C Versions.” Prentice Hall
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