Unit 4

Three-Dimensional Geometric Transformation

3D computer graphics or three dimensional computer graphics are graphics that use a three-dimensional representation of geometric data that is stored in the computer for the purpose of performing calculations and rendering 2D images.

2D is "flat" using the horizontal and vertical (X & Y) dimensions, the image has only two dimensions. 3D adds the depth (Z) dimension. This third dimension allows for rotation and visualization from multiple perspectives.

We can perform different transformation by specifying the three dimensional transformation vector, however the 3D transformation is more complex than 2D transformation.

Q. *What are the issue in 3D that makes it more complex than 2D?*

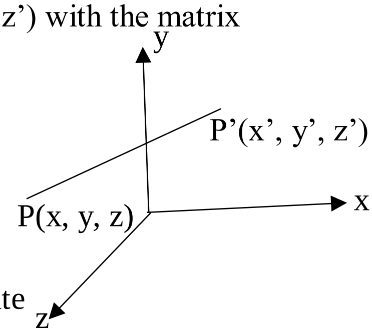
→When we model and display a three-dimensional scene, there are many more considerations we must take into account besides just including coordinate value as 2D, some of them are:

- Relatively more coordinates point are necessary.
- Object boundaries can be constructed with various combinations of plane and curved surfaces.
- Consideration of projection (dimension change with distance) and transparency.
- Many consideration on visible surface detection and remove the hidden surfaces.

➢ 3D Geometric Transformation

▪ 3D Translation

A point is translated from position P(x, y, z) to position P'(x', y', z') with the matrix operation as:



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Parameter $t_x,\ t_y, t_z$ specify transition distances for the coordinate directions x, y, z. This matrix representation is equivalent to three equations:

$$x' = x + t_x$$

$$y' = y + t_y$$

$$z' = z + t_z$$

- **3D Rotation**

Rotation about Z-axis

$x' = x cos\theta - y sin\theta$

$y' = x sin\theta + y cos\theta$

$z' = z$

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

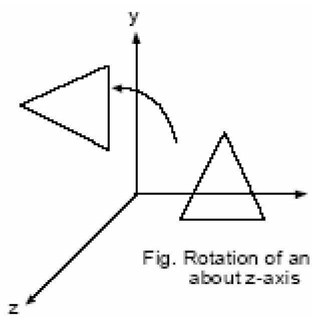


Fig. Rotation of an object about z-axis

Rotation about x-axis

$x' = x$

$y' = y cos\theta - z sin\theta$

$z' = y sin\theta + z cos\theta$

$$\therefore R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

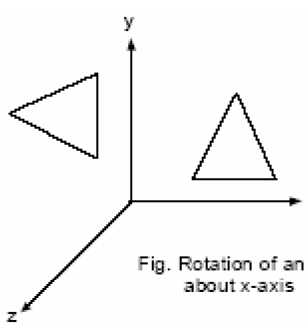


Fig. Rotation of an object about x-axis

Rotation about y-axis

$y' = y$

$z' = z cos\theta - x sin\theta$

$x' = z\sin\theta + x\cos\theta$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

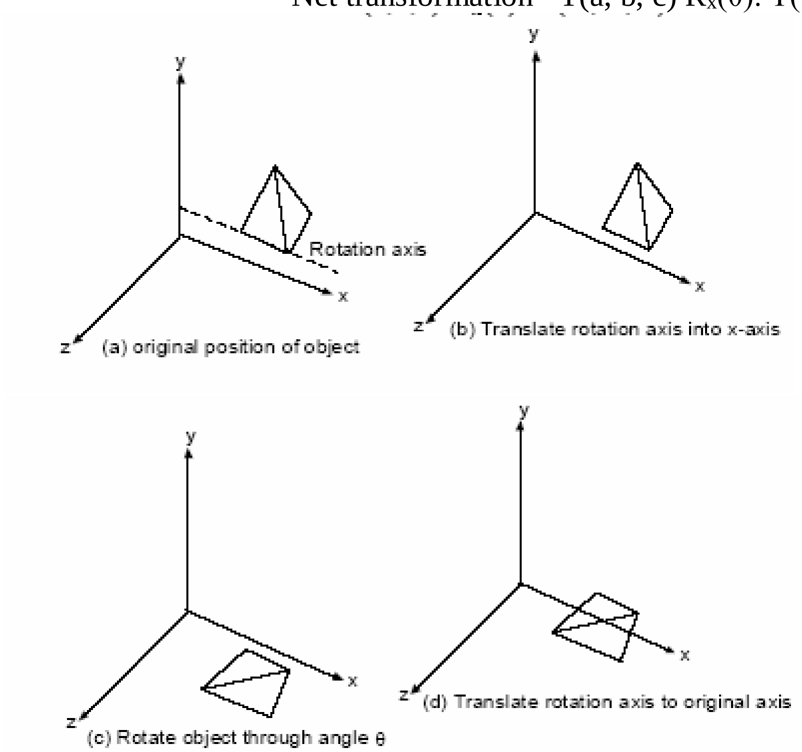
#General 3D rotation (Rotation about any coordinate axis)

➢ ***Parallel to any of the co-axis:***

When an object is to be rotated about an axis that is parallel to one of the co-ordinate axis, we need to perform some series of transformation.

1. Translate the object so that the rotation axis coincides with the parallel co-ordinate axis. T(-a,-b,-c) where, (a, b, c) is any point on the rotation axis.
2. Performed the specified rotation about the axis. $R_x(\theta)$
3. Translate the object so that the rotation axis is moved to its original position. T(a, b, c).

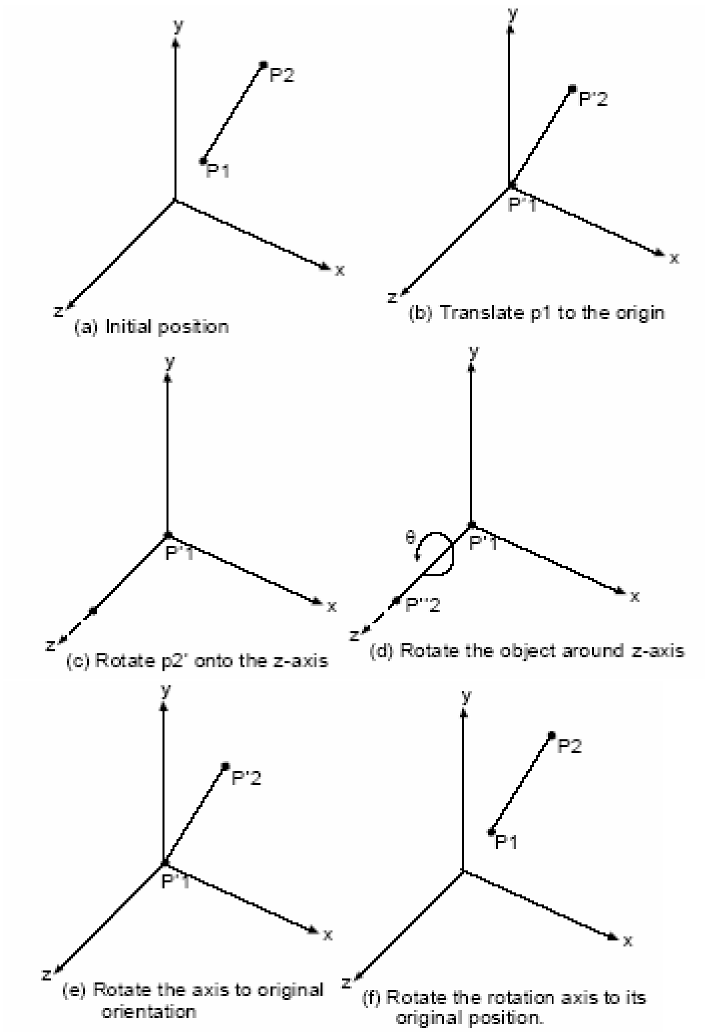
Net transformation= T(a, b, c) $R_x(\theta)$. T(-a,-b,-c)



➢ ***Not parallel to any of the co-axis:***

When an object is to be rotated about an axis that is not parallel to one of the co-ordinate axes, we need to perform some series of transformation.

1. Translate the object such that rotation axis passes through co-ordinate origin.
2. Rotate the object such that axis of rotation coincides with one of the co-ordinate axis.
3. Perform the specific rotation about selected coordinate axis.
4. Apply inverse rotation to bring the rotation axis back to its original orientation.
5. Apply inverse translation to bring the rotation axis back to its original position.



(a) Initial position
(b) Translate p1 to the origin
(c) Rotate p2' onto the z-axis
(d) Rotate the object around z-axis
(e) Rotate the axis to original orientation
(f) Rotate the rotation axis to its original position.

1) Translate P1 to origin

$$T(-x_1,-y_1,-z_1) = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

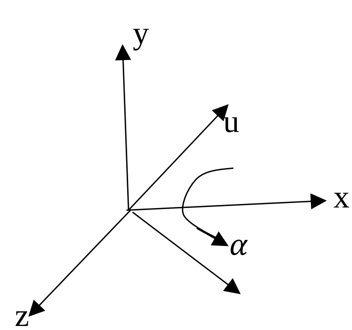
2) Find transformation that will bring rotation axis P1P2 on z-axis. This will be accomplished in two steps:

a) Rotation by angle α about x axis that bring vector $\vec{u}$ into x-z plane, where,

$$\vec{v} = \overrightarrow{p_2} - \overrightarrow{p_1} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

Unit vector along $\vec{v}$, $\vec{u} = \frac{\vec{v}}{|\vec{v}|} = (a, b, c)$ where,

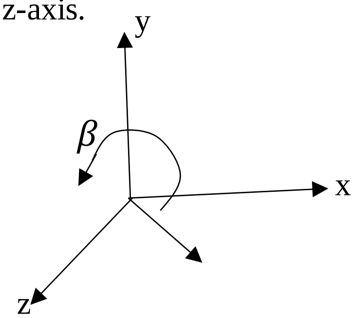
$a = \frac{x_2 - x_1}{|\vec{v}|}$, $b = \frac{y_2 - y_1}{|\vec{v}|}$, $c = \frac{z_2 - z_1}{|\vec{v}|}$



$$\therefore R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c/d & -b/d & 0 \\ 0 & b/d & c/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where, $d = \sqrt{b^2 + c^2}$

b) Rotate by angle β about y axis that brings $\vec{u}$ on z-axis.



$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} d & 0 & -a & 0 \\ 0 & 1 & 0 & 0 \\ a & 0 & d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3) Rotation about z-axis.

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4) Find the $R_y^{-1}(\beta)$, $R_x^{-1}(\alpha)$, T^{-1}i.e. $T(x_1, y_1, z_1)$.

5) Find composite transformation for general rotation by 'θ' (anticlockwise).

$$M = T(x_1, y_1, z_1).R_x^{-1}(\alpha).R_y^{-1}(\beta).R_z(\theta).R_y(\beta).R_x(\alpha).T(-x_1, -y_1, -z_1)$$

Q. Find the new co-ordinates of a unit cube 90 degree rotated about an axis defined by its end points A(2, 1, 0) and B(3, 3, 1).

Solution:

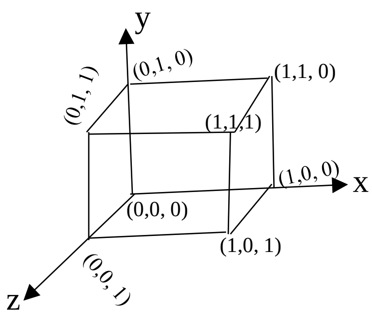


Fig: unit cube

Now,
Translating the point (A) to the origin,

$$T = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, rotate the $A'B'$ about x-axis by angle α until vector $\vec{u}$ lies on xz-plane. Where,
$\vec{v}$ = $\vec{B}$ - $\vec{A}$ =(3, 3, 1)-(2, 1, 0)=(1, 2, 1)

Unit vector along $\vec{v}$, $\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{(1,\ 2,\ 1)}{\sqrt{6}} = (\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}})$ = (a, b, c) (say)

And, $d = \sqrt{b^2 + c^2} = \sqrt{\frac{5}{6}}$

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\alpha & -sin\alpha & 0 \\ 0 & sin\alpha & cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c/d & -b/d & 0 \\ 0 & b/d & c/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{5} & -2/\sqrt{5} & 0 \\ 0 & 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Again, rotating $A'B'$ about y-axis by angle β until it coincides with z-axis.

$$R_y(\beta) = \begin{bmatrix} cos\beta & 0 & sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -sin\beta & 0 & cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} d & 0 & -a & 0 \\ 0 & 1 & 0 & 0 \\ a & 0 & d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{5/6} & 0 & -1/\sqrt{6} & 0 \\ 0 & 1 & 0 & 0 \\ 1/\sqrt{6} & 0 & \sqrt{5/6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotating the unit cube 90 degree about z-axis.

$$R_z(90^0) = \begin{bmatrix} cos90^0 & -sin90^0 & 0 & 0 \\ sin90^0 & cos90^0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The combined transformation rotation matrix about the arbitrary axis becomes,

$R(\theta) = T^{-1}R_x^{-1}(\alpha).R_y^{-1}(\beta).R_z(90^0).R_y(\beta).R_x(\alpha).T$

$$R(\theta) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & -2/\sqrt{5} & 1/\sqrt{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{5/6} & 0 & 1/\sqrt{6} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{6} & 0 & \sqrt{5/6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{5/6} & 0 & -1/\sqrt{6} & 0 \\ 0 & 1 & 0 & 0 \\ 1/\sqrt{6} & 0 & \sqrt{5/6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{5} & -2/\sqrt{5} & 0 \\ 0 & 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore R(\theta) = \begin{bmatrix} 0.116 & -0.075 & 0.983 & 1.742 \\ 0.742 & 0.667 & 0.075 & -1.151 \\ -0.650 & 0.741 & 0.167 & 0.560 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, multiplying $R(\theta)$ by the matrix of original unit cube;

$P' = R(\theta).P$

$$P' = \begin{bmatrix} 0.116 & -0.075 & 0.983 & 1.742 \\ 0.742 & 0.667 & 0.075 & -1.151 \\ -0.650 & 0.741 & 0.167 & 0.560 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 2.650 & 1.667 & 1.834 & 2.816 & 2.7525 & 1.742 & 1.909 & 2.891 \\ -0.558 & -0.484 & 0.258 & 0.184 & -1.225 & -1.152 & -0.409 & -0.483 \\ 1.467 & 1.301 & 0.650 & 0.817 & 0.726 & 0.566 & -0.091 & 0.076 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- **3D Scaling**

Matrix representation for scaling transformation of a position $P = (x, y, z)$ relative to the coordinate origin can be written as;

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

→For scaling of point P(x, y, z) w.r.t to fixed point (x_f, y_f, z_f) can be represented with the following transformation.

1. Translate the fixed point to the origin. $T(-x_f, -y_f, -z_f)$
2. Apply scaling w.r.to origin. $S(s_x, s_y, s_z)$
3. Translate the fixed point back to its original position. $T(x_f, y_f, z_f)$

$$= \begin{bmatrix} 1 & 0 & 0 & x_f \\ 0 & 1 & 0 & y_f \\ 0 & 0 & 1 & z_f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_f \\ 0 & 1 & 0 & -y_f \\ 0 & 0 & 1 & -z_f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} s_x & 0 & 0 & (1-s_x)x_f \\ 0 & s_y & 0 & (1-s_y)y_f \\ 0 & 0 & s_z & (1-s_z)z_f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **3D Reflection**

In 3D-reflection the reflection takes place about a plane.

(a) About xy-plane (z-axis)

This transformation changes the sign of the z coordinates, leaving the x and y coordinate values unchanged.

$$R_{fxy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) About xz-plane (y-axis)

This transformation changes the sign of the y coordinates, leaving the x and z coordinate values unchanged.

$$R_{fxz} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) About yz-plane (x-axis)

This transformation changes the sign of the x coordinates, leaving the y and z coordinate values unchanged.

$$R_{fyz} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

About any plane:

Step-1: Translate the plane such that it passes through origin i.e. normal vector passes through origin.
Step-2: Rotate the plane such that normal vector lies on one of the co-ordinate axis.
Step-3: Perform reflection about the plane whose normal vector is one of the co-ordinate axis.
Step-4: Rotate back the plane such that normal vector takes its original orientation.
Step-5: Translate back the plane to its original position.

- **<u>3D Shearing</u>**

Shearing transformations are used to modify objects shape.

(a) Z-axis shearing

This transformation alters x and y coordinates values by amount that is proportional to the z values while leaving z coordinate unchanged.

$x' = x + s_{hx}z$
$y' = y + s_{hy}z$
$z' = z$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & s_{hx} & 0 \\ 0 & 1 & s_{hy} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(b) X-axis shearing

This transformation alters y and z coordinates values by amount that is proportional to the x values while leaving x- coordinate unchanged.

$x' = x$
$y' = y + s_{hy}x$
$z' = z + s_{hz}x$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ s_{hy} & 1 & 0 & 0 \\ s_{hz} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(c) Y- axis shearing

This transformation alters x and z coordinates values by amount that is proportional to the y values while leaving y- coordinate unchanged.

$x' = x + s_{hx}y$
$y' = y$
$z' = z + s_{hz}y$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & s_{hx} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & s_{hz} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Q. A homogenous coordinate point P(3, 2, 1) is translated in x, y, z direction by -2, -2 & -2 unit respectively followed by successive rotation of 60⁰ about x- axis. Find the final position of homogenous coordinate.

Solution:
Here,
$t_x = -2$
$t_y = -2$
$t_z = -2$

$$T = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x(60^0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos60 & -sin60 & 0 \\ 0 & sin60 & cos60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Composite transformation

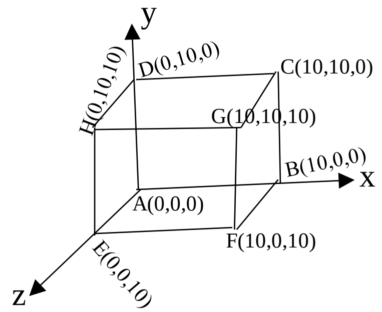
$$R_x(60^0).T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & -1+\sqrt{3} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & -\sqrt{3}-1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now,

$$P' = M.P = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & -1+\sqrt{3} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & -\sqrt{3}-1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix} =$$

Q. A cube of length 10 units having one of its corner at origin (0, 0, 0) & three edges along principal axis. If the cube is to be rotated about z-axis by an angle of 45^0 in counter clockwise direction, calculate the new position of point.

Solution:



$$R_z(45^0) = \begin{bmatrix} cos45 & -sin45 & 0 & 0 \\ sin45 & cos45 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.707 & -0.707 & 0 & 0 \\ 0.707 & 0.707 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now,

$$\begin{bmatrix} 0.707 & -0.707 & 0 & 0 \\ 0.707 & 0.707 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \begin{matrix} A & B & C & D & E & F & G & H \end{matrix} \\ \begin{bmatrix} 0 & 10 & 10 & 0 & 0 & 10 & 10 & 0 \\ 0 & 0 & 10 & 10 & 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & 0 & 10 & 10 & 10 & 10 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array} =$$

(Complete yourself)

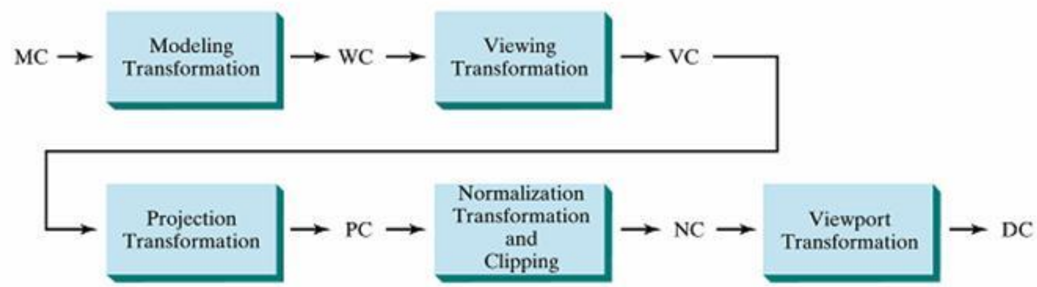
❖ <u>3D Viewing</u>

In 3D viewing, we specify a view volume in the world, a projection onto a projection plane, and a viewport on the view surface. Conceptually, objects in 3D world are clipped against the 3D view volume and are then projected. The contents of the projection of the view volume onto the projection plane, called the window, are then transformed (mapped) into the viewport for display.

Viewing in 3D involves the following considerations:

- We can view an object from any spatial position. E.g. In front of an object, Behind the object, In the middle of a group of the objects, Inside an object.
- 3D descriptions of objects must be projected onto the flat viewing surface of the output device.
- The clipping boundaries enclose a volume of space.

3D Viewing Pipeline



General three-dimensional transformation pipeline, from modeling coordinates to world coordinates to viewing coordinates to projection coordinates to normalized coordinates and, ultimately, to device coordinates.

Modeling transformation and viewing transformation can be done by 3D transformations. The viewing-coordinate system is used in graphics package as a reference for specifying the observer viewing position and the position of the projection plane. Projection operations convert the viewing-coordinate description (3D) to coordinate position on the projection plane (2D). (Usually combined with clipping, visual-surface identification, and surface rendering). Normalization transformation & clipping and view port transformation maps the coordinate positions on the projection plane to the output device.

❖ **Projection**

- Projection is any method of mapping three dimensional (3D) objects into two dimensional (2D) view plane (screen). In general, projection transforms a N-dimension points to N-1 dimensions.
- Two types of projection:

  a) Parallel projection:

  In parallel projection, coordinate positions are transformed to view plane along parallel line.

  - A parallel projection preserves relative proportion of objects so that accurate views of various sides of an object are obtained but doesn't give realistic representation of the 3D objects.
  - Can be used for exact measurement so parallel lines remain parallel.

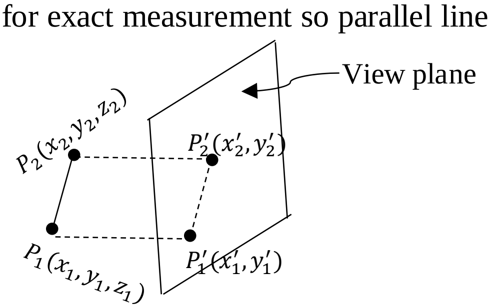


Fig: Parallel projection of an object to the view plane

b) *Perspective projection:*

In perspective projection, objects positions are transformed to the view plane along lines that converge to point behind view plane.

- A perspective projection produces realistic views but does not preserve relative proportions. Projections of distance objects from view plane are smaller than the projections of objects of the same size that are closer to the projection place.

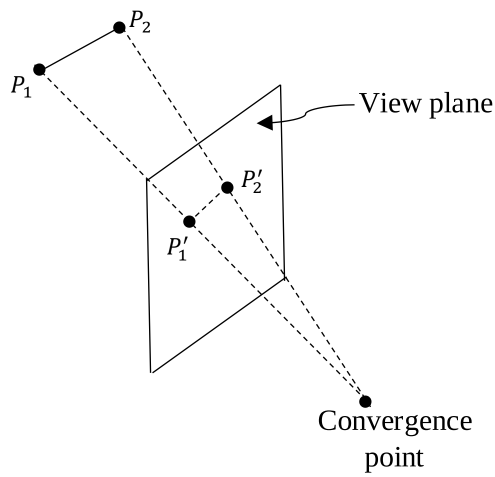


Fig: Perspective projection of an object to the view plane

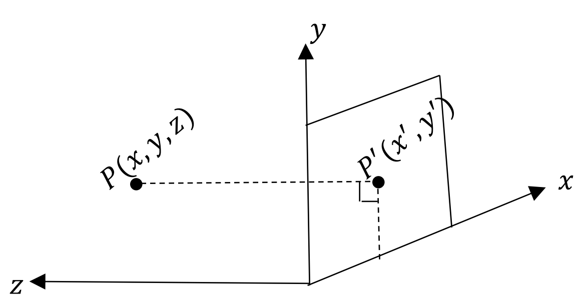
➢ ***Parallel projection***

On the basis of angle made by projection line with view plane, there are two types of parallel projection.

i. Orthographic parallel projection
ii. Oblique parallel projection

i. Orthographic parallel projection

- When the projection lines are perpendicular to view plane, the projection is orthographic parallel projection.



Here, after projection of $P(x, y, z)$ on XY-plane we get $P'(x', y')$ where,

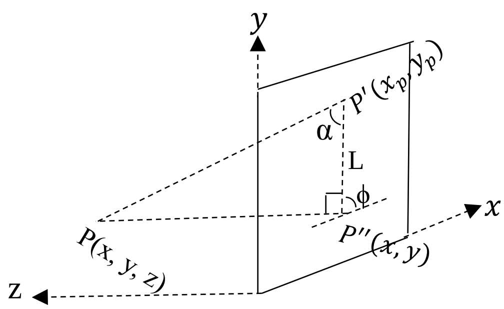
$x' = x$, $y' = y$ & z=0

In homogenous coordinate form,

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

ii. *Oblique parallel projection*

- Here, projection lines make certain angle to view plane.
- Oblique projection is specified with two angle ‘α’ & ‘ϕ’ where ‘α’ is the angle made by projection line with view plane line (L) which is formed by joining oblique projected point $P'(x_p, y_p)$ and orthogonal projected point $P''(x, y)$ on point (x, y) on view plane & ‘ϕ’ is the angle between ‘L’ & horizontal direction of view plane.



The projected point $P'(x_p, y_p)$ on XY plane is given by,

$x_p = x + L\cos\theta$

$y_p = y + L\sin\theta$

Since $\tan \propto = \frac{z}{L}$

$\therefore x_p = x + z\cot\alpha \cos\phi$

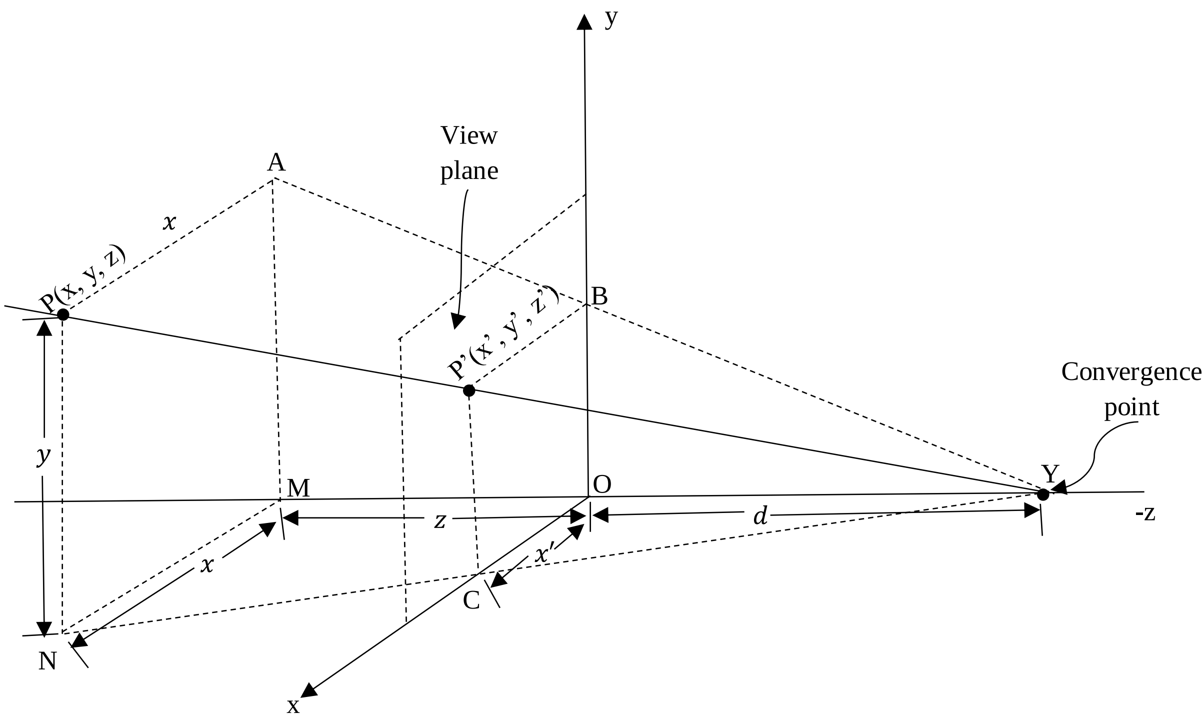
$y_p = y + z\cot\alpha \sin\phi$

In homogenous matrix form

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cot\alpha\cos\phi & 0 \\ 0 & 1 & \cot\alpha\sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

➢ ***Perspective projection***

Let $P(x, y, z)$ be projected on X-Y view plane by perspective projection at $P'(x', y', z')$ & projected line converges point 'y' an z-axis at is distance 'd' from XY view plane.



We have to find the value of $P'(x', y', z')$ from similar triangle NMY & COY.

$$\frac{NM}{MY} = \frac{CO}{OY}$$

$$\frac{x}{z+d} = \frac{x'}{d}$$

$$\therefore\ x' = x\frac{d}{z+d}$$

Similarly, from similar triangle AMY & BOY

$$\frac{AM}{MY} = \frac{BO}{BY}$$

$$\frac{y}{z+d} = \frac{y'}{d}$$

$$\therefore\ y' = y\frac{d}{z+d}$$

Difference between Parallel and Perspective Projection

Parallel Projection	Perspective Projection
Coordinate position of object are transferred into view plane along parallel line.	Coordinate positions are transferred into view plane along lines that converges to a point called convergence point.
Relative proportion of object are maintained.	Relative position not maintained.
It gives accurate view of object.	If view plane is nearest to object image appear larger & if view plane is farther image appear smaller.
It doesn't give realistic view of object.	It gives more realistic view of object as perceived by human eye.
Used in engineering and architectural drawing.	Used in building design, rail track design, whose realistic view is needed.

References

- **Donald Hearne and M.Pauline Baker**, "Computer Graphics, C Versions." Prentice Hall